Mass Anisotropy Searching for the Effects in a Foucault Pendulum

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Introduction

In his Principia, Newton wrote about his bucket experiment [1]. He filled a bucket with water and tied a piece of string around the handle. He then spun the bucket slightly and showed that the water in the bucket did not move much. He then twisted the string and allowed the bucket to spin as the string uncurled, and noticed that the water assumed a parabolic shape once the motion of the bucket had been communicated to the water in the bucket. His conclusion was that so long as the water was not moving relative to absolute space then there was no effect on the water. Newton defines, "Absolute space is in its own nature without relation to anything external, remains always similar and immovable." Once the water began to follow the bucket and hence have motion in absolute space then the water would assume the parabolic shape.

In the 1800's Ernest Mach approached Newton's experiment differently. If we imagine the mass distribution spinning about the bucket we can see no physical difference between this scenario and Newton's bucket experiment. So perhaps the shape of the water is caused by something other than absolute motion. The mass of the universe may be able to affect the inertia of the water.

Using Cocconi and Salpeter's Work

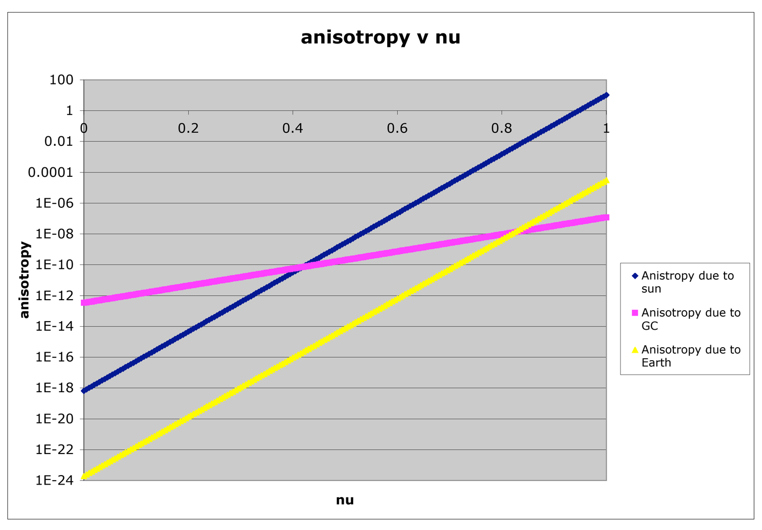
Cocconi and Salpeter gave a formulation of Mach's ideas [2]. They began with the assumptions:

- Inertial mass is proportional to the mass distribution of the universe around the object. Hence any non-uniformities in the mass distribution in the universe could lead to *anisotropic mass.*
- Mass anisotropy, Δ M

$$\Delta M \propto \frac{M}{r^{\nu}}$$

M is the mass of a massive object impacting the anisotropy, and r is the vector distance away.

- Using the above formula we see that ν lies between 0 and 0.4. It is ensured that objects further away from the Earth than the Sun have a larger effect on the Anisotropy, than the mass of Earth and the Sun.



Anisotropy Due to Nearby Massive Objects as a Function of ν

This anisotropy is then directly related to the mass at the Galactic Center.

- This contribution depends on the angle θ between r and the acceleration of the test body.
- The vector pointing towards the Galactic Center defines a preferred direction. The most attractive dependence, which maintains energy conservation, has a maximum value $\cos(\theta) = 1$. The angular dependence between acceleration and the preferred direction transforms Newton's Law into:

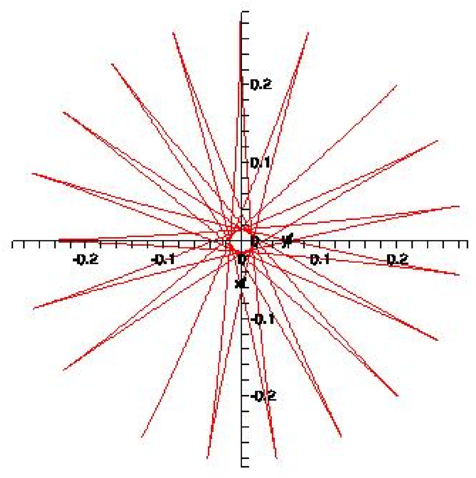
$$F_i = M_{ij} a_j$$

Cocconi and Salpeter tell us what they believe the entries for M_{ij} should be. Using basic Linear Algebra we can find the entries for the surface of the Earth rotating relative to the Galactic Center, the preferred direction.

A Foucault Pendulum

The Foucault Pendulum

A Foucault Pendulum is similar to a typical pendulum however the top of the string is free to rotate. What we see is that the pendulum bob goes to relative maxima and minima, and while so doing it also rotates relative to the rotation of the Earth. In this figure the string's length has been adjusted to illustrate the path of the Foucault pendulum as seen from above.



A Top View of the Path of a Foucault Pendulum

Coordinate System

We will be using Cartesian coordinates for the following calculations. We choose our coordinates such that the $\hat{q}_3$ axis points away from the surface of he Earth, the $\hat{q}_2$ axis is perpendicular to the $\hat{q}_3$ axis and points toward the North Pole, and the $\hat{q}_1$ axis is mutually orthogonal.

Review of Classical Mechanics

We first write the Lagrangian for the Original Foucault Pendulum. Because we are on the surface of a rotating planet we are in a non-inertial reference frame, thus there will be a Coriolis Effect which is proportional to $\vec{\omega} \times \vec{v}$ [3]. We know that:

$$L = \frac{m}{2}(\dot{q_1}^2 + \dot{q_2}^2 + 2(\varepsilon^{ijk}\omega_j q_k)\dot{q}_i) - mgq_3$$

By using a small angle approximation we may write q_3 as $\frac{q_1^2+q_2^2}{2l}$. We now use the Lagrangian, L, to find the Hamiltonian. By definition we know that $p_l = \frac{\partial L}{\partial \dot{x}_l}$. Which implies that $p_1 = m\dot{q}_1 - m\omega q_2 \sin(\lambda)$ and $p_2 = m\dot{q}_2 + m\omega q_1 \sin(\lambda)$. Thus the Hamiltonian is:

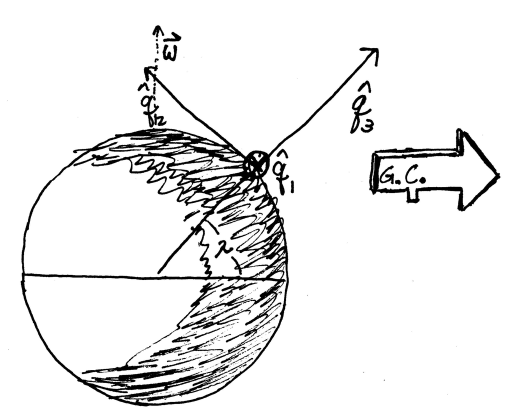
$$H = \frac{p_1^2 + p_1^2}{2m} + p_1\varepsilon^{123}q_2\omega_3 + p_2\varepsilon^{213}q_1\omega_3 + mg\frac{q_1^2+q_2^2}{2l}$$

Which result in the equations of motion:

$$\ddot{q}_1 = -2\varepsilon^{123}\omega_3\dot{q}_2 + \frac{gq_1}{l}$$
$$\ddot{q}_2 = 2\varepsilon^{213}\omega_3\dot{q}_1 + \frac{gq_2}{l}$$

with solutions

$$q_1 = \cos(\sin(\lambda)\omega t)\cos(\sqrt{\frac{g}{l}}t)$$
$$q_2 = \sin(\sin(\lambda)\omega t)\cos(\sqrt{\frac{g}{l}}t).$$



Our Coordinate System

Foucault Pendulum with Mass Anisotropy

Suppose we were to consider anisotropic mass in the context of a Foucault Pendulum. In a non-inertial reference frame M_{ij} is a function of time, $M_{ij} = M_{ij}(t)$.

$$H = \frac{p_j p_n M_{nj}^{-1}}{2} - M_{ij}\epsilon^{jkl}\omega^k q^l p_n M_{ni}^{-1} + M_{33}g_3\frac{q_1^2+q_2^2}{2l}$$

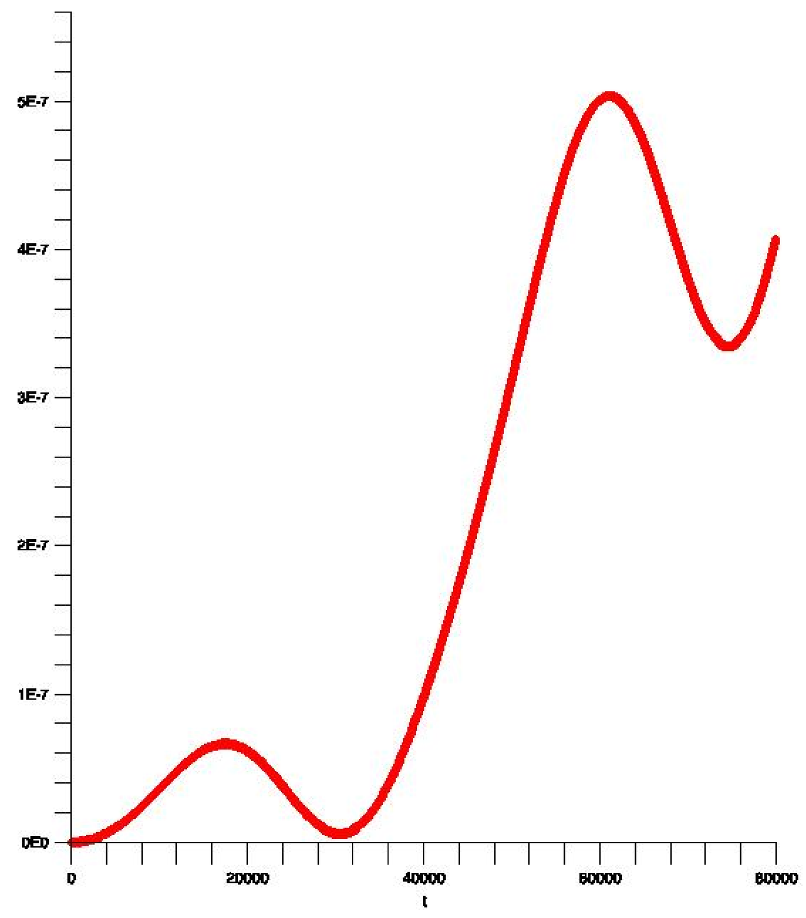
Which yield the following equations of motion:

$$M_{bj}\ddot{q}_j + \dot{M}_{bj}\dot{q}_j + 2M_{bj}\epsilon^{jkl}\omega^k\dot{q}^l + \frac{M_{33}g_3q_b}{l}$$
$$= -\dot{M}_{bj}\dot{q}_j - \dot{M}_{bj}\epsilon^{jkl}\omega^k q^l$$

These equations of motion are the same as the Isotropic Mass Foucault Pendulum but there are two additional terms the $-2\dot{M}_{bj}\dot{q}_j - \dot{M}_{bj}\epsilon^{jkl}\omega^k q^l$. These two terms serve to drive the Anisotropic Foucault Pendulum and have an interesting result.

Results

The path of the anisotropic mass and isotropic mass Foucault pendulum look nearly identical. However, if we compare the angular frequencies we can see a difference:



The Difference in Precession Frequencies between Anisotropic and Isotropic mass Foucault Pendulum

This graph shows the Anisotropic Foucualt Pendulum Frequency less the Isotropic Foucualt Pendulum Frequency. We notice there is a very small change in frequency but look at the relative maxima and minima. If we take the time distance between two maxima or minima we notice there is a difference of about 44,000 seconds, or a half sidereal day. Twice a day the mass of the pendulum is increased and its precession slows down, and another two times a day the mass is decreased and the pendulum frequency is increased.

Conclusion and extensions

In the anisotropic case we observe that the pendulum is being driven relative to its orientation toward the Galactic Center. The anisotropic mass is less than the isotropic mass when $\hat{q}_1$ is perpendicular to the Galactic Center, and the anisotropic is greater than the isotropic mass when $\hat{q}_2$ is parallel or anti-parallel to the Galactic Center. The driving of the angular frequency is proportional to the latitude of the Earth. The driving is greatest at the North/South Pole and the least at the Equator. The effects of mass anisotropy can help in the search of finding discrete space time.

Acknowledgements

We are grateful to Hamilton College, and to the Hamilton College Physics Department for help and advice.

References

[1] Newton, I. Principia Mathematics.
[2] Cocconi, G., Salpeter, E. *Nuovo Cimento* **X** (1958) 3608.
[3] Hand, L., Finch, J Analytical Mechanics Cambridge, 1998.