1. False. If preferences are time inconsistent, then discounting is applied inconsistently over time. For example, if a person discounts the future at a constant rate (i.e., has a constant degree of impatience), then there is no time inconsistency. However, if a person always has extra impatience in the present, then he will feel that waiting (delaying gratification) now is more unpleasant than it will be in the future. However, he will feel the same way tomorrow (hence the inconsistency). If he recognizes the time inconsistency, then he can solve the problem rationally (i.e., accurately maximize his utility), whereas if he does not recognize the inconsistency, he may fall into the trap of ongoing procrastination.

2. To see whether the project is profitable, calculate

\[ PV_{\text{benefits}} = \frac{.38}{1.1} + \frac{.38}{(1.1)^2} + \frac{.38}{(1.1)^3} \approx .95 < 1 = PV_{\text{costs}} \]

so don’t take the project. We would be better off investing our money in a bond than in this project, or equivalently would lose money if we borrowed to finance the project.

To see why we needed to do this present value calculation, note that the firm gets a rate of return of 0.38 for 3 years, which is better than that on a bond, but 0 thereafter, which is worse. The sum of the returns is 3 \times 0.38 = $1.14, which is greater than the price of the machinery ($1), but we shouldn’t value $0.38 received three years from now as highly as $0.38 today, since to get this we need only put \[ \frac{0.38}{(1.1)^3} = $0.29 \] in a bond today. We can receive the exact stream of payments that we get from the project over the three years by putting only $0.95 into a bond instead of putting the $1 into the project.

3. The answer is that the firm should not purchase the equipment. The interest rate would have to be less than 5% for the firm to find the investment profitable.

To get this answer, we need to work out the present value calculation for the firm. First we can make the following useful observation: If the interest rate \( i \) is constant over time, the PV of a constant perpetual flow of $x per period, that starts next period, is \( \frac{x}{1 + i} \). Let’s derive this:

\[
PV = \frac{x}{1+i} + \frac{x}{(1+i)^2} + \frac{x}{(1+i)^3} + ... \\
= \frac{x}{1+i} \times (1 + \frac{1}{1+i} + (\frac{1}{1+i})^2 + (\frac{1}{1+i})^3 + ...) \\
= \frac{x}{1+i} \times \frac{1}{1-\frac{1}{1+i}} \\
= \frac{x}{i}
\]

Now back to the problem. Suppose that the firm finances the equipment out of cash on hand. Then the cost of purchasing and running the equipment is the $100,000 outlay today plus the future maintenance costs of $5000 per year. The PV of these maintenance costs is

\[
PV_{\text{maintenance costs}} = \frac{5000}{1.07} + \frac{5000}{(1.07)^2} + \frac{5000}{(1.07)^3} + ... \\
= \frac{5000}{0.07} \\
\approx 71,400
\]

Thus, the PV of the costs of buying and operating the equipment is $100,000 + $71,400 = $171,400.
The benefit of operating the equipment is $10,000 per year in increased revenues. The PV of this flow of benefits is

\[ PV_{\text{benefits}} = \frac{10,000}{1.07} + \frac{10,000}{(1.07)^2} + \frac{10,000}{(1.07)^3} + \ldots \]

\[ = \frac{10,000}{0.07} \approx 142,800 \]

Since the PV of the benefits is less than the PV of the costs, you should not purchase the machine.

At what interest rate would this project be profitable? The PV of the benefits is \( 10,000 / i \) and the PV of the costs is \( 100,000 + 5000 / i \). Thus, for the PV of the benefits to be greater than the PV of the cost we need

\[ \frac{10,000}{i} > 100,000 + \frac{5000}{i} \]

which will be true if

\[ \frac{5,000}{i} > 100,000 \]

and thus if

\[ i < \frac{5,000}{100,000} = .05 \]

If the interest rate on bonds was less than 5%, the investment would be profitable.\(^2\)

Other Investment Models: Mankiw presents a per period capital rental rule for investment rather than the present value rule that we are working with. Since, in the current problem, the flow of benefits and costs over time are constant, the two rules turn out to be equivalent in this case. However, in problem two above, the returns varied over time (we get benefits in three years only). Since the investment is irreversible and has different returns in different years, we can’t decide period by period whether to make the investment – we have to look at the entire sequence of expected future costs and benefits. Thus, in that case, the PV rule works, whereas Mankiw’s period by period rule would not. The PV rule is the more general rule.

It should also be that the Tobin’s Q rule would give a similar result as the PV rule. Tobin’s Q is the ratio of the stock market’s valuation of the firm’s capital stock to the replacement cost of that capital stock. If Q is greater than one, then stock holders anticipate that this firm will generate more profits in the future than can be earned on bonds (of similar risk). If Q is less than one, then stock holders view this firm as unprofitable relative to bonds. In the latter case, and if new investments by the firm are perceived by the stock market to be no more profitable than the firm’s past investments, then you should not purchase the equipment, because doing so will drive down the price per share of your company’s stock further. The central difference between the PV rule and the Tobin’s Q rule is that Tobin’s Q uses the stock market investor’s expectations of future benefits and costs rather than your’s (i.e., rather than the company managers’ expectations) to evaluate the expected profitability of investment projects.

4.

a. The capital stock is currently

\[ K = K^* = \frac{Y}{r + d} = \frac{2000}{0.05} = 40,000 \]

Since 3% of this depreciates each period, gross investment \( (I) \) must be \( 0.03 K = 1200 \).

\(^2\) If the interest rate on bonds was exactly 5%, we would be indifferent between investing in the project and investing in bonds. This interest rate 5% is sometimes called the “internal rate of return” of the project and is a measure of the marginal net benefit of the project expressed as a rate of return.
b. If \( Y \) rises to 2100, then the desired capital stock in the economy rises to 42,000. To achieve this, firms must have net investment \( I_n \) this year of 2000, which requires gross investment of

\[
I = I_n + dK = 2000 + 1200 = 3200
\]

However, in the following year, and all subsequent years, gross investment falls back to the new (slightly higher) level of annual depreciation, \( 0.03 \times 42,000 = 1260 \), which is just enough to maintain the capital stock at its desired level. Notice that \( I \) is much more sensitive to changes in \( Y \) (the accelerator effect) than to the level of \( Y \) (look at the graph above: \( \Delta Y \) is temporary but produces a large increase in \( I \), whereas the level of \( Y \) remains high, but has a small impact on \( I \)).

This analysis has assumed that GDP remains fixed at 2100. As a result of the short run rise in investment spending, we would actually expect GDP to rise further. This would raise desired capital stocks further, and thus boost investment further. Notice that the accelerator effect is destabilizing for \( Y \) (makes \( Y \) fluctuations larger).

c. The desired capital stock falls to \( 2100/0.06 = 35,000 \). Once firms have adjusted their capital stocks downward, maintaining them will require investment of \( 0.04 \times 35,000 = 1400 \) which is greater than the prior level. Thus, in this example, though the rise in the rate of depreciation reduces the desired (and thus actual) capital stock, it increases the total amount of depreciation per year, causing gross investment spending \( I \) to rise in the long run.

Note that depreciation rates have been rising in the U.S. in the past 30 years (equipment has become less durable on average, especially due to the increasing use of computers). Some of the increase in investment spending during that time is simply due to the fact that computers become obsolete very rapidly (have very high depreciation rates) and so businesses are now spending more money replacing them than they did with more durable plant and equipment.

5. The investment tax credit, once implemented, will reduce the effective purchase cost of equipment. If businesses get a tax credit worth a fraction \( \tau \) of their purchases of capital goods, then the effective cost of such goods with price \( P_K \) becomes \( (1 - \tau) P_K \). This should have the effect of stimulating investment spending.

However, if businesses expect the ITC to be implemented next year, they may wait to buy capital goods until the credit is in place, rather than buying now at the higher effective cost. Thus, we may actually see investment spending fall this year, following the campaign announcement, making the recession temporarily deeper.

6. a. Here, \( m = \frac{cr+1}{rr+1} = \frac{cr+1}{cr+1} = 1 \). with \( rr = 1 \) banks don’t lend out any of their deposits. If the Fed increases \( B \) by $1, it will be divided by the public between C and D according to the value of \( cr \), but banks will hold all of the increased D as R, so there will be no further expansion of deposits through bank loans. The money supply rises by $1.

b. \( cr \) is \( \infty \) so \( m = 1 \). If the Fed increases \( B \) by $1, the public holds it all as currency, so deposits do not rise, and banks have no new deposits to lend out. Money supply rises by $1.

c. \( m = \frac{cr+1}{rr+1} = 1.56 \), so \( M = 166,666 \). If \( rr \) rises to 0.25, \( m \) falls to 1.6, so that \( M \) falls to 160,000. Banks hold deposits as reserves, bank loans, and other assets such as bonds. To increase the ratio of reserves to
deposits, they must call in bank loans (or sell some of their bond holdings) and convert these to reserves. This causes deposit withdrawals from other banks and a subsequent chain of deposit destruction.

7.

a. The nominal money supply $M$ is

\[ M = m \times B \]
\[ = 2 \times 1200 \]
\[ = 2400 \]

and the price level $P$ is 1. Equating real money supply $(M/P)$ and money demand, we get $Y = 2400/0.8 = 3000$.

Notice that since LM is vertical, equilibrium output $Y$ is determined solely by LM, but interest rates $r$ depend also on spending (IS).

b. The Fed is increasing the monetary base $B$ by 100. Since the money multiplier is 2, this increases the money supply by 200. Equating the new money supply (2600) with money demand, we get $Y = 3250$. Output increases by 250.

The money supply expands by the increase in the base (100) plus a further increase (100) through the process of deposit creation via banks loans. This expansion of the money supply depresses interest rates, stimulating investment spending, and thus causing output to increase.

c. Now we have an increase in the monetary base by 100 as above but also an increase in government purchases by 100. If the K-Cross multiplier is 5, then IS shifts to the right by 500 (i.e., spending rises by 500 if interest rates stay constant). Interest rates will rise, however, unless the money supply rises sufficiently (i.e., LM is shifted out sufficiently) to offset the pressure of increased spending on interest rates. But LM has only shifted to the right by 250, so interest rates do increase, though not by as much as if the increase in the deficit was not monetized (i.e., if the Fed left $M$ constant).