This example set of problems contains more problems than the mid-term but it does not contain a complete listing of the types of problems which might appear on the mid-term. Aside from trying these problems, please review the problem sets, class notes, and your text. There is the beginnings of an equation sheet on the second page.

This is a 2 hour, closed-book exam held under the auspices of the Hamilton Honor Code. All work must be your own. Show all your work. Useful information is included on the Handy Relations page.

Problems:

1. A solid sphere is charged with radially dependent density \( \rho = Cr^2 \) where \( C \) is a constant. Find the \( E \)-field inside and outside the sphere.

2. An infinite sheet has a uniform surface charge density \( \sigma \). Find the \( E \)-field everywhere. Solve for the \( E \)-field around two sheets of opposite charge separated by a distance \( d \).

3. Find the electric potential \( \phi(z) \) of a disk with uniform surface charge density \( \sigma \) and radius \( a \). Find an expression for the field when \( z > a \) and check that it conforms to what you expect.

4. In class we found the potential inside a slot with constant potential \( \phi_0 \) on the inside wall at \( x = 0 \). Find the potential \( \phi(x,y) \) for the same slot except the wall at \( x = 0 \) consists of many metal strips which yield a potential \( \phi_0(y) = Cy \) on \( y = 0 \) to \( y = a \).

5. Find the electric field of a charge \( q \) positioned a distance \( h \) above a conducting plate. Find the surface charge distribution on the plate.

6. Derive the transformations of the electric field

\[ E'_\perp = \gamma E_\perp \quad \text{and} \quad E'_\parallel = E_\parallel \]

using two carefully chosen capacitors.

7. A battery, capacitor, and resistor are connected in series around a loop. Initially the there is no charge on the capacitor. What is the charge at time \( t \)?

8. You have a 1/2 lb. spool of No. 28 copper wire (0.32 mm) used for driving a pendulum. The length of wire has a resistance of \( R = 210 \Omega \). What is the length of the wire? Assume a resistivity of \( 1.7 \times 10^{-8} \Omega \text{ m} \).
Handy Relations

General:
The Taylor series of a function \( f(x) \) around \( x = 0 \) is
\[
f(x) = f(0) + \frac{df}{dx}\bigg|_{x=0} x + \frac{1}{2} \frac{d^2 f}{dx^2}\bigg|_{x=0} x^2 + \frac{1}{6} \frac{d^3 f}{dx^3}\bigg|_{x=0} x^3 + \ldots
\]
\[
\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}
\]
\[
(1 + x)^n \simeq 1 + nx
\]
\[
t' = \gamma \left( t - \frac{vx}{c^2} \right)
\]
\[
x' = \gamma (x - vt)
\]
\[
y' = y
\]
\[
z' = z
\]

\( E \) and \( B \) Fields:
\[
E = -\nabla \phi
\]
\[
F = q(E + \mathbf{v} \times B)
\]
\[
E = \frac{F}{q} = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2}
\]
\[
\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2, \quad \mu_0 = 4\pi \times 10^{-7} \text{Tm/A}
\]
\[
c = \omega/k = 1/\sqrt{\epsilon_0 \mu_0}
\]
\[
E'_{\perp} = \gamma E_{\perp} \quad \text{and} \quad E'_{\parallel} = E_{\parallel}
\]
\[
U = \frac{\epsilon_0}{2} \int E^2 \, dv
\]
\[
I = \int \mathbf{J} \cdot da \quad \text{and} \quad \nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0
\]
\[
Q = CV, \quad V = IR, \quad P = IV, \quad C = \frac{\epsilon_0 A}{d}
\]
\[
R = \frac{\rho L}{A}
\]
\[
\phi = \frac{1}{4\pi \epsilon_0} \frac{q}{r} \quad \text{and} \quad \phi_b - \phi_a = -\int_a^b \mathbf{E} \cdot ds
\]
\[
\int \mathbf{B} \cdot ds = \mu_0 I_{\text{encl}} B = \frac{\mu_0 I}{2\pi r}
\]
\[
\tau = \mu \times B, \quad \mu = IA
\]
\[
\Phi_B = \int \mathbf{B} \cdot da
\]
\[
F = I\ell \times \mathbf{B}
\]
\[
\phi(x,y) = \sum_{n=1}^{\infty} b_n e^{-n\pi x/a} \sin(n\pi y/a)
\]
\[
b_n = \frac{2}{a} \int_0^a \phi(y) \sin(n\pi y/a) \, dy