So you know the theory already:

\[
\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} = 0 \\
\n\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \\
\mathbf{F} = q \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right)
\]

For the rest of the semester we unravel many consequences of this theory.

In class we have jogged our memories of the vector calculus used in Maxwell’s equations. Chapter 1 has vector algebra and calculus, a bit on curvilinear coordinates, the Dirac delta function, and even a section on vectors fields in general. I added an introduction to index notation for vectors, including \( \epsilon_{ijk} \). I like the notation a lot since it simultaneously means that I memorize (or look up) fewer identities and can avoid the serious awkward features of the usual notation, which we bump into when we discuss magnetic forces, for instance. But there is a bit of steep learning curve. I’d like to try this approach and then use the notation with which you are most comfortable.

Much of the initial material is review. Keep me posted on the best pace for the seminar.

**Problems: (Due on Friday August 31)**

1. 1.2 Write the two expressions in index notation.
2. 1.4 Cross product and normal practice
3. 1.10 Understand what a pseudo-X is.
4. 1.12 Working with the gradient and a “hill”.
5. 1.13 We started this and we’ll use it later (that’s what the dot means). Start a formula sheet and include this one.
6. 1.22( a) and (b) Use index notation.
7. Prove product rule (ii). Use index notation. You will see what I mean about the awkward bits in the usual notation.
8. 1.37 Playing with spherical coordinates
9. 1.38 Playing with unit vectors in spherical coordinates
10. 1.45 Working with \( \delta() \)
11. 1.46
12. 1.47 The first part of the answers are all Dirac \( \delta \)-functions.
13. 1.62 Needed later

In this first week on math review you will present problem solutions. Next week we will add presentations on material. Everyone should work on all the problems but the listing below gives the folks who are primarily responsible for complete solutions.

**Presentations:** Be prepared to present and/or answer questions on the following problems

- Hannah 1, 6, 11
- Seth 2, 7, 12
- Abby 3, 8, 13
- Katie 4, 9
- RJ 5, 10
Notes on text:

• **Read the “Advertisement”:** It is lovely and Griffiths places electrodynamics in the larger picture. We will return to this at the end of the course as well. Any idea why Griffiths leaves out “the other dimension” or realm of mechanics, general relativity? Another question to ponder is this: If electromagnetic forces are the “dominant ones in everyday life” then why does gravity determine the large scale structure of the universe (including the existence of a space for electromagnetic forces to act!)? Griffiths does not mention the “range of validity” of electrodynamics, i.e. when we can reasonably expect the theory to give sensible answers. The best theories are those which tell us in what regimes they are valid. Any ideas on deciding when the classical electrodynamical description is adequate?

- **Chapter 1:** Perhaps you are familiar which the Kronneker delta $\delta_{ij}$ or the antisymmetric (also known as the Levi-Civita symbol), $\epsilon_{ijk}$. The scalar or dot product is $v \cdot w = \sum_i v^i w_i$ - just like the definition in component form. The “epsilon” is completely antisymmetric in pairs of indices so that, e.g. $\epsilon_{123} = 1$ while $\epsilon_{213} = -1$. It provides another way of working with vector products. For instance, a cross product for a vector $A$ (or, equivalently $A^\ell$) is

$$ (A \times B)^i = \epsilon^{ijk} A^j B^k $$

(The position of the indices is only critical when working in more general spaces; vectors have indices “upstairs.”) The triple cross product is written as

$$ (A \times (B \times C))^m = \epsilon^{m\ell i} A^\ell \epsilon_{ijk} B^j C^k. $$

How would one write the triple product $A \cdot (B \times C)$? For more complicated products the identity

$$ \epsilon^{ijk} \epsilon_{\ell m} = \epsilon^{ijk} \epsilon_{\ell m} = \delta_i^\ell \delta^j_m - \delta_i^j \delta^\ell_m $$

is useful.

• **page 8** A way of remembering the “BAC-CAB” rule for $A \times (B \times C)$: The first term of the expansion contains the middle vector with the scalar coefficient being the dot product of the other two vectors. The second term carries a negative sign and contains the “last vector” (the other one inside the parentheses). The scalar coefficient is the dot product of the remaining two vectors.

If you have been sleeping through the first sections wake up a bit for the bit on Griffiths script conventions. He uses them extensively.

• **page 10** As good as this book is there are some mild faults. In section 1.1.5 Griffiths makes the point, “If it transforms like a vector, it is a vector.” The (subtle) problem with this approach is that vectors exist independently of any components or coordinate systems. (Griffiths does point this out in the footnote on page 39.) This way of defining vectors is the “old way.” As those who took GR know the way to get started on the general definition is via the gradient, discussed in 1.2.2, and curves.

In more detail, a curve $\gamma(t)$ is a function which gives a point in the space if it is fed a number (the parameter value of $t$). At a point on the curve, a variation in the parameter $t$, $d/dt$, yields a magnitude and a direction - a vector! This variation also, of course, is linear and satisfies the product rule. So we define a vector at this point as the derivative. Why is this “nice”? First, there is no mention of coordinates. Second, it does not use displacements over finite separations (bad because in general spaces, “a vector here is not the same as the vector

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1 This is not easy! In a similar situation, Einstein and Grossmann missed one condition for determining the classical limit of general relativity and this cost Einstein five years to straighten out!

2 Beware that my $ijk$ index notation is not the same as Griffiths’ in this chapter. He uses these as in $v_a$ is “another” $v$ vector than $v_b$, rather than my use which is different components. Unfortunately his notation is not upwards compatible.
over there”). Third, this definition emphasizes that the derivatives generate a “motion” along a curve as one would expect for a tangent vector. By the way we can still keep our old notions of vectors (at least at a point) by choosing the curves to be the usual coordinate frame.

- page 13 - The “heart” of the chapter. This is what we need to understand the Maxwell equations. Gain fluency by running through problems 1.11, 1.12, 1.15, and 1.18. Griffiths has great “hands on” descriptions of divergence and curl. Try giving three dimensional examples. There is an explosion of product rules in section 1.2.6 and 1.2.7. Take these slowly. Understand each one (what do the symbols mean? where does it come from? does it seem reasonable? where can we use it? does the order of factors matter?)

- page 21 Product rules galore! With the the \( \epsilon \) notation you can re-derive these at a drop of a hat.

- page 24 Section 1.3 is the second major subject of any calculus – integrals. Griffiths builds them up studying integrals in 1, 2, and 3 dimensions before turning to their relations (Green’s, Stokes’, and divergence theorems). He labors through several examples. If this material is at all rusty for you, these examples are a gold mine. Use them! (Work the problem without looking at the solution. When you run into the answer or a problem, greet it, and return to the example to check your solution with Griffiths’.) Check your understanding with a subset of 1.18, 1.29, and 1.30. The theorems have an overall structure: They relate integrals in \( n + 1 \) dimensions to ones in \( n \) dimensions, e.g. 1 and 0 (which is the fundamental theorem of calculus). By the way, the idea of equating the integration over a volume (in any dimension) to its boundary is surprising and deep.

- page 38: Carefully work through all this (up to Eq. (1.69)) so that you can recreate it at a moment’s notice. We will have to several times this semester.

- page 45: The Dirac delta “function” \( \delta(x) \) is not a function but a distribution. A raw, mathematically imprecise, definition is “distributions can only be evaluated inside integrals.” If you end a calculation with the RHS of Eq. (1.88), for example, this is bad news. The result might as well be \( \infty \). Nevertheless, the formulae Griffiths sprinkles throughout this chapter are useful. Do a couple of the integrals on page 49. Deltas make integration easy!

- page 50: Important material about the 3D Dirac delta that we use over and over again.