Here we want to illustrate how cooperation can be a viable outcome in an indefinitely repeated prisoners' dilemma game.

In a one shot prisoners' dilemma, there is a unique Nash equilibrium in which both players defect. This is an inefficient outcome from the point of view of the players, as both would be better off if both cooperated. However, cooperation is strictly dominated by defection for each player.

This result carries over to a finitely repeated prisoners' dilemma game. Each player might consider whether cooperation early in the game can be maintained to the players' mutual advantage. However, applying backward induction, we find that the only subgame perfect Nash equilibrium (SPNE) of the finitely repeated game is for each player to always defect. The Nash equilibrium of the last round of the game (treated in isolation) is for each player to defect. Consequently, the Nash equilibrium starting in the next to last round, given mutual defection in the final round, is mutual defection, and so on. A player using this logic of backward induction will not expect that cooperation in earlier rounds will induce cooperation in later rounds.

The result does not, however, carry over to infinitely repeated games or games with an uncertain number of rounds. In these cases, it may be possible to elicit cooperation in any given round from your opponent by cooperating now and threatening to defect in future rounds if your opponent defects now. Such strategies are called trigger strategies, the most famous being tit-for-tat. These strategies reward cooperation and punish defection and can be consistent with backward induction (i.e., can be subgame perfect) because there is no fixed final period in which defection is expected to be played; the players expect in each period that the game will (or at least may) continue to be played into the future.

Discounting: To see how this can work, we first need to determine how players evaluate an infinite stream of future payoffs. We assume that players *discount* future payoffs. I.e., they place less value on a payoff received in the future than they would place on the same payoff received today. Specifically, we assume that the utility currently placed on a payoff x to be received t periods from now is $\delta^t x$ and that the *discount factor* $\delta \in [0, 1)$. If payoffs are monetary, we typically assume that $\delta = \frac{1}{1+r}$, where r is the interest rate at which the player can borrow or lend. In that case, $\delta^t x$ is the amount of money that we can borrow today against the future payoff x, and we call $\delta^t x$ the *present value* of x.

Geometric Series: The following results about geometric series will come in handy. For $\delta \in [0, 1)$,

$$1 + \delta + \delta^2 + \delta^3 + ... = \frac{1}{1 - \delta}$$

Consequently,

$$\delta + \delta^2 + \delta^3 + ... = \delta \cdot (1 + \delta + \delta^2 + \delta^3 + ...)$$
$$= \frac{\delta}{1 - \delta}$$

Example: Consider the prisoners' dilemma with the payoffs that we have used previously (e.g., 2x2 games handout). Suppose that two players play this game (with simultaneous moves, as usual) once per period, viewing the outcome and receiving payoffs at the end of each round. Suppose further that Player Two plays tit-for-tat, cooperating in the first round and then matching the last move of player one in subsequent rounds.

How will Player One want to respond to this strategy? It turns out that, if the discount factor is high enough (i.e., if the players value the future enough), then tit-for-tat is a best reply of Player One to this strategy. Further, both players playing tit-for-tat is a SPNE of the game.

Here we will show a related but more modest claim: that if the discount factor is high enough, Player One would prefer to cooperate in all rounds than to defect in all rounds, given that Player Two plays tit-for-tat.

Given that Player Two plays tit-for-tat, the present value of the payoff stream to Player One of cooperating in every round is

$$V_C = 2 + \delta \cdot 2 + \delta^2 \cdot 2 + \delta^3 \cdot 2 + ... = \frac{2}{1 - \delta}$$

The payoff to Player One of defecting in every round is

$$V_D = 3 + \delta \cdot 1 + \delta^2 \cdot 1 + \delta^3 \cdot 1 + ... = 3 + 1 \cdot \frac{\delta}{1 - \delta}$$

Thus, Player One prefers cooperating in all rounds to defecting in all rounds if

$$\frac{2}{1 - \delta} > 3 + 1 \cdot \frac{\delta}{1 - \delta}$$

i.e., if

$$\delta > \frac{1}{2}$$

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Thus, if the discount factor δ is greater than $\frac{1}{2}$, then tit-for-tat played by player 2 can elicit cooperation from player one, under the assumption that player one selects between always cooperating and always defecting. As noted above, we can go on to show that cooperating in all periods is indeed a best response for player one among all strategies in her strategy space, as is tit-for-tat.