Spacetime and the Quantum World: Guide 2

Fall 2025 v1.0

1. Reading: For this assignment

Mermin, *It's About Time* Chapters 4 - 6. I prefer to work with simultaneity (Ch 5) before velocity addition (Ch 4) partly just to give us more time with this most peculiar of aspects of special relativity. Ch 6 has time dilation and length contraction. Mermin also has an (idiosyncratic) introduction to space-time diagrams in chapter 10.

(Optional but helpful!) **Ellis and Williams**, *Flat and Curved Space-times*
Chapter 2 discusses the operational definitions of measuring location and time using light. A pdf of this is on the Blackboard page for Phys 135. The book is also on reserve in the Library.

Chapter 3 covers many of the unusual effects of special relativity are worked out here using space-time diagrams. Sections 3.1 - 3.3 will be most relevant for this guide, although we will skip pages 58-9. Sections 3.4 (time dilation) and 3.5 (length contraction) give a new perspective on these results.

2. Reading: Looking ahead

Mermin, *It's About Time* Chapters 7 and 8.

Ellis and Williams, *Flat and Curved Space-times*
Section 3.6, (the whole kit and kaboodle) will (probably) be discussed the week of September 23. Some of Chapter 3 is available on Blackboard. The book should also be on reserve in the Library.

3. Questions: Due Thursday, September 25 by 11 PM.

Please submit your solutions using Gradescope (code ZY635K). (You can also get to this through Blackboard: 135 - Tools - Gradescope).

(1) Two, identical (excepting paint schemes), elastic balls collide. Before the collision, the blue ball on the right moves to the left at 18 ms^{-1} while the buff ball on the left is stationary.
   (a) Using the switching between reference frames method determine the outcome of the collision in the original frame.
   (b) Make a space-time diagram of the collision in the original frame.
   (c) Make a space-time diagram of the collision in the frame of the known situation.

(2) Two volcanoes, Mt. Boom and Mt. Doom, are 500 km apart in their rest frame. Suppose that each erupts in a burst of light. An observer in a lab halfway between the two volcanoes receives the light from the two blasts at the same time. The above objects (mountains, observer, and assistant) are at rest with respect to each other. We found that in this frame all observers agree that the eruptions occurred simultaneously. (If you don't agree, let's talk.)

   A spacecraft flying by at 4/5 c from Mt. Boom to Mt. Doom is 50 km from Mt. Boom and 450 km from Mt. Doom when they erupt, in the mountains' frame.
   (a) According to an observer on the spacecraft does the eruption at Mt. Boom occur before, at the same time, or after the eruption at Mt. Doom?
   (b) Draw a space-time diagram of the events in the spacecraft's frame.
   (c) If you found that the eruptions are not simultaneous in the spacecraft's frame then compute the time delay between the events.

(d) What would change if the spacecraft was going the other way, i e. from Mt. Doom to Mt. Boom?

Assume the mountains and observers are all on a single line. You can also neglect any non-inertial effects due to being on the surface of the Earth.

(3) The difference in times between different frames is very small for everyday speeds. Even time aboard the International Space Station differs only by a millionth of a second during an hour of time on the ground. As the speed of the clock increases, the effect grows. Show that while our clock ticks off one hour,

(a) a clock moving at half the speed of light ticks off 52 minutes and
(b) a clock moving at 3/5 the speed of light ticks off 48 minutes.

(4) In Carlo Rovelli's book "Essential Relativity" he writes, "... given two events happening in different locations, it is meaningless to say that two events happen 'at the same time t', unless we specify ..." What do we have to specify? Give an example.

(5) (2 pts.) A super-fast WorldStar train moves uniformly during a thunderstorm. Lighting bolts strike the front and back of the train, leaving char marks on the ground. Sophie is a dedicated trainspotter, in macintosh, standing directly next to the tracks on the embankment watching the train pass. She sees both flashes of light from the lighting strikes in the same instant. Theodore is half-way down the car looking out of the window at the storm. He sees Sophie as she passes by and the two flashes of light at the same moment.

(a) Sketch the situation in Sophie's reference frame, just as the light flashes reach Sophie. Include the light fronts, Sophie, the char marks on the ground, and the train. Indicate the direction of motion of the train with an arrow.
(b) Again in Sophie's reference frame, sketch the situation shortly after the lighting strikes. This is *before* your previous picture. Please include the light fronts, Sophie, the char marks on the ground, Theodore, and the train.
(c) Now switch to Theodore's reference frame, and sketch the situation just as the light flashes reach Theodore, i.e. repeat part (a) in Theodore's frame. Please be sure include the direction of Sophie's motion.
(d) In Theodore's reference frame, sketch the situation shortly after the lighting has struck both ends of the train. This is the analog to part (b) in Theodore's frame. Include the light fronts, Sophie (with direction arrow), the char marks (with direction arrows), and the train.
(e) Describe in words the order of the events of the lighting strikes and observations of the flashes of light according to Sophie.
(f) Describe in words the order of the events of the lighting strikes and observations of the flashes of light according to Theodore.

Hint: Recall that all observers must agree on the existence of events. For instance if two flashes of light reach an observer at the same moment - defining an event - then all observers will agree that this particular event occurred, as defined by the arrival of the flashed of light at the observer. However, all observers do not have to agree on the *order* of distant events.

(6) Suppose a 'mind reader' in London claims to know what his twin brother in New Zealand says at any moment, within less than one-hundredth of a second after a word is uttered. Is there anything extraordinary about this claim?

(7) *Space-time diagrams* Harold captains a rocket moving at $v = \frac{1}{2}c$ in the $+x$ direction relative to Maud, who is on Mars. Their positions coincide at $t = 0$.

(a) Plot the worldlines of Harold and Maud in a (t, x) space-time diagram, in Maud's frame.

(b) The rocket emits light in the forward and backward directions at $t = 2s$. Add the light fronts to the diagram.

(c) Maud signals to the rocket at $t = 1s$. What is the earliest time she can expect a reply?

(8) Marcus, on a rocket, moves away from Ovid to the left at $-\frac{3}{4}c$, and Cattallus, also on a shiny new rocket, moves away from Ovid (poor Ovid) to the right at $\frac{3}{4}c$.

(a) Draw a spacetime diagram of this history in Ovid's frame.

(b) What is the rate of increase of the separation between Marcus and Cattallus in Ovid's frame? You can use your space-time diagram.

(9) The Hafele-Keating experiment – Taking clocks on a trip round the world. An experimental test of the "twin paradox" was done in October 1971 with two atomic clocks. J. Hafele and R. Keating took atomic clocks, one eastward, one westward, around the world twice using commercial airlines. When they returned they compared the their times with the clock at the US Naval Observatory. A typical passenger aircraft speed is 540 mph or about 254 m/s.

(a) Compute the value of

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

at this speed by using a calculator. This is a tiny effect! Your calculator may not report many enough digits to say something different than "1". Hint: Don't work hard at this, simply type in the numbers and record the calculator's response.

(b) Let's use the very handy approximation for γ[1]

$$\gamma \simeq 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2$$

Compute the "leading order" correction, $\frac{1}{2}\left(\frac{v}{c}\right)^2$ for the aircraft speed.

(c) Assume that one leg of the flight took 14 hours. On this leg of the trip, by how much did the traveling clock disagree with the clock that stayed "home"? Please state your results in seconds.

Interestingly, Hafele and Keating had to account for general relativistic effects as well. With these effects, they found agreement between their experiment and the theoretical predictions.

(10) (2 pts.) A Federation cruiser is at rest relative to the border of Klingon Space and is in Federation Space. In the Federation cruiser's frame the border is 6 light-hours away. A Klingon battleship zips by the cruiser moving at $\frac{3}{5}c$ towards the border. Let's call this event A and have it designate noon for both frames. A little while later the Klingon battle ship fires a parting shot in the form of a laser beam ("phaser") at the Federation cruiser. The phaser impacts the cruiser at 8 PM, according to the Federation clocks, severely damaging the spaceship.

(a) Sketch a space-time diagram of the history in the cruiser's frame, including the spaceships, border, and phaser.

(b) According to the Federation cruiser's clocks, when does the Klingon ship pass into Klingon Space? Explain how you found this result.

[1]If you are curious about this approximation you can use the nifty approximation (or Taylor series)

$$(1 - x)^a \approx 1 - ax$$

with $a = -\frac{1}{2}$ and $|x| \ll 1$. If you are skeptical about this relation try it out for a few examples.

(c) According to the Federation clocks, when did the Klingon battleship fire the phaser? Explain.

(d) What is the times for the events after the ships pass according to the Federation cruiser?

(e) Describe the history of events in the Klingon's frame.

(f) Many months later the case comes to Intergalatic Court. The Klingon-Federation Space-time Treaty states that it is illegal for a Klingon (Federation) ship in Federation (Klingon) territory to damage Federation (Klingon) property. The lawyer representing the Klingon ship's captain argues that they are within the letter of the law, since in the ship's frame the damage to the cruiser occurred after the Klingon ship crossed back into Klingon territory. Hence they were not in Federation territory at the time the damage occurred. Did the event of the phaser impacting the cruiser really occur after the Klingon ship crossed into Klingon territory, in their frame?

(g) On the basis of this case, would you recommend that the Intergalatic Congress re-negotiate the treaty to clarify this law? If so, how would you recommend wording the treaty? If not, what is your advice to ship captain's?