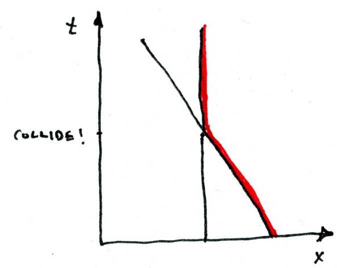
1. Solutions

(1) Two, identical elastic balls collide.

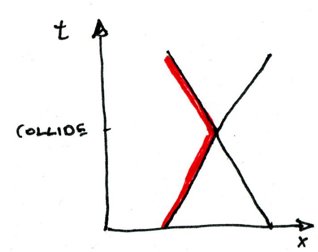
(a) Since the known situation is one in which the objects approach with equal speed, we should switch to a frame moving left at 9 ms^{-1} (or a velocity of -9 ms^{-1}). In this frame the objects recoil at equal speeds, here 9 ms^{-1}. To find the outcome of the collision in the original frame we switch back by moving to the right at 9 ms^{-1}. The result is that the buff ball moves at 18 ms^{-1} to the left and the blue ball is stationary.

(b) Other than paint scheme here's a spacetime diagram of the collision in the original frame



(To fix the colors red → blue and black → buff.)

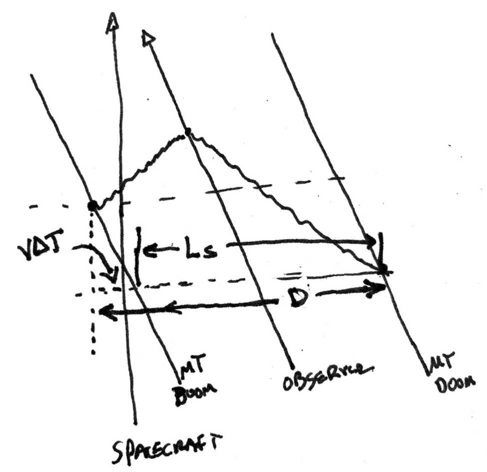
(c) And a spacetime diagram of the collision in the frame of the known situation is



(2) Mt. Boom and Mt. Doom return:

(a) Key elements include: The light from the eruptions arrives simultaneously at the observer. Since the ground frame is moving relative to the spacecraft and since the speed of light is constant then the two eruptions occur at different times in this frame; Mt. Doom erupts first. On the line that connects the events of the eruptions the spacecraft is to the right of Mt. Boom and to the left of Mt. Doom; this is the representation of the 50 km and 450 km mentioned in the problem.

(b) Here's a space-time diagram of the events in the spacecraft's frame:



I have added distances in this frame for the computation in part (b).

(c) There are a number of ways to solve this. Here's my current favorite solution: From the diagram we can see that

$$D = v\Delta T + L_S$$

where L_S is the distance between the mountains in the spacecraft frame and D is the distance between the eruptions. The slip in simultaneity gives

$$\Delta T = \frac{vD}{c^2} \text{ or } D = \frac{c^2 \Delta T}{v}.$$

Equating these two expressions for D and collecting terms gives

$$\left(\frac{c^2}{v}\right)\left(1 - \frac{v^2}{c^2}\right)\Delta T = L_S.$$

Recognizing γ and solving for ΔT gives

$$\Delta T = \frac{v\gamma^2 L_S}{c^2}.$$

Now, due to length contraction, the distances between the mountains is

$$L_S = \frac{L_M}{\gamma},$$

where L_M is the distance in the mountains' frame, 500 km. This implies

$$\Delta T = \frac{v\gamma L_M}{c^2} = \frac{\frac{4}{5}c\frac{5}{3}500 \text{ km}}{c^2} = \frac{2}{9} \times 10^{-2} \text{ s} \simeq 2.2 \text{ ms}.$$

(d) If the spacecraft was going from Mt. Doom to Mt. Boom then the slip in simultaneity would be the other way around; Mt. Boom would erupt before Mt. Doom. In the spacetime diagram the worldlines of the mountains and observer would slant the other way and the order of the eruptions would reverse. The delay would be the same.

(3) Computing time dilation from $t = \gamma t'$. While our clock ticks off one hour, a clock moving at half the speed of light ticks off t' or

$$1 = \gamma t' \implies t' = 1/\gamma = \frac{1}{1.15} \simeq 0.87 \text{ hr} \simeq 52 \text{ min. where } \gamma = \frac{1}{\sqrt{1 - \frac{1}{4}}} \simeq 1.15.$$

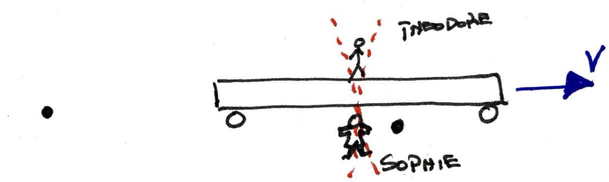
Likewise, a clock moving at 3/5 the speed of light ticks off 48 minutes since

$$t' = 1/\gamma = \frac{4}{5} = 0.8 \text{ hr } = 48 \text{ min. where } \gamma = \frac{1}{\sqrt{1 - \frac{9}{25}}} = \frac{5}{4}.$$

(4) Because two simultaneous events in one reference frame are not simultaneous in any other frame, the rest of Carlo Rovelli's sentence could be "... a reference frame" or "...an observer." For instance, the full quote could be, '... given two events happening in different locations, it is meaningless to say that two events happen 'at the same time t', unless we specify a reference frame."

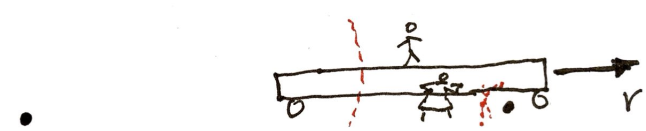
(5) (2 pts.) The super-fast WorldStar train:

(a) In Sophie's reference frame the WorldStar train passes by, say moving to the right. Here's the moment when the light fronts reach Sophie.



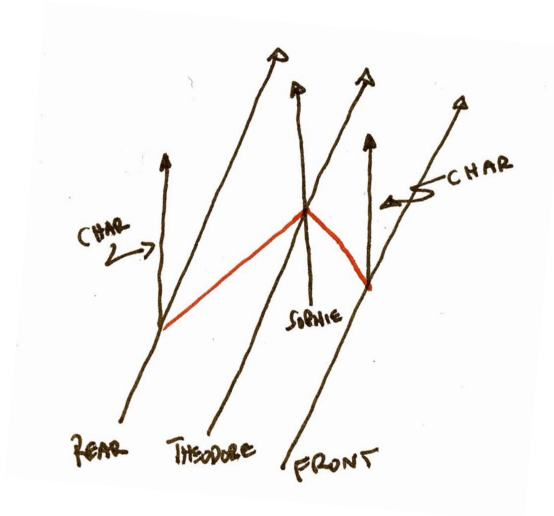
I found it helpful to draw these diagrams with the analogous spacetime diagrams, which I include at the end.
Key elements include light fronts arriving at Sophie and Theodore and the char marks on the ground displaced to the rear since the light took some time to arrive while the trains moved.

(b) Here's the situation shortly after the lighting strikes.

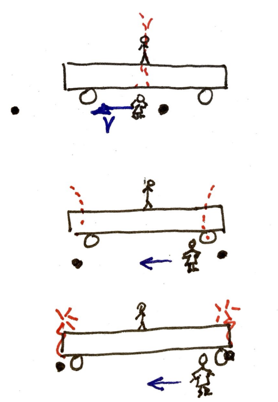


Notice how the light fronts are equidistant from Sophie but are of different distances from the char marks. This occurs because the events are not simultaneous in Sophie's frame.

Here's the spacetime diagram of the events in Sophie's frame



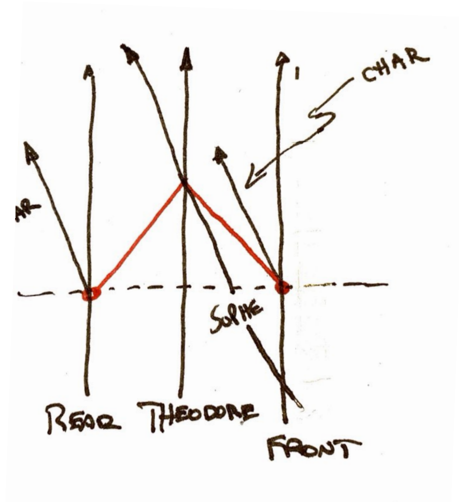
(c) Here's a sketch of the history in Theodore's reference frame, with time running up the diagram



The top diagram is the answer to this part.

(d) This is the middle drawing above.

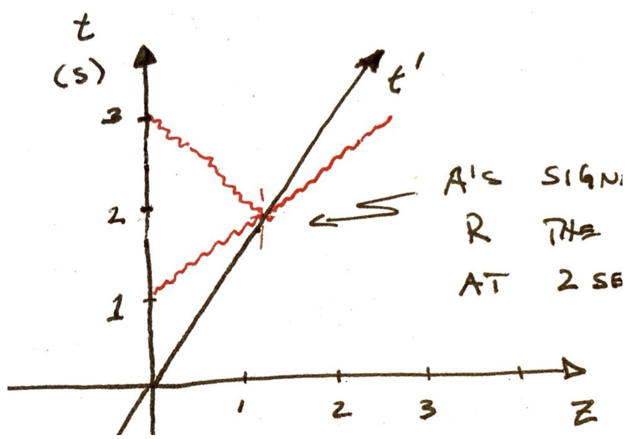
Here's the spacetime diagram of the events in Theodore's frame



(e) In Sophie's frame the rear of the train in struck by lighting first. The front is struck next. Finally the flashes of light (or light fronts) are seen by Sophie.
(f) In Theodore's frame the light strikes both ends of the train simultaneously and then Theodore sees the flashes of light, which is also the same event when Sophie sees the flashes.

(6) The earth has a radius of about 6400 km. So light or a radio signal would take $\pi r/c \simeq \pi(6400 \text{ km})/3 \times 10^8 \simeq 0.067$ s. (Even if it passed through the earth somehow, light would take $d/c \simeq 0.04$ s to make the trip.) Thus, if the signal is light-like it could not take less time than this. So there is reason to be skeptical about the 'less than one-hundredth of a second' claim.

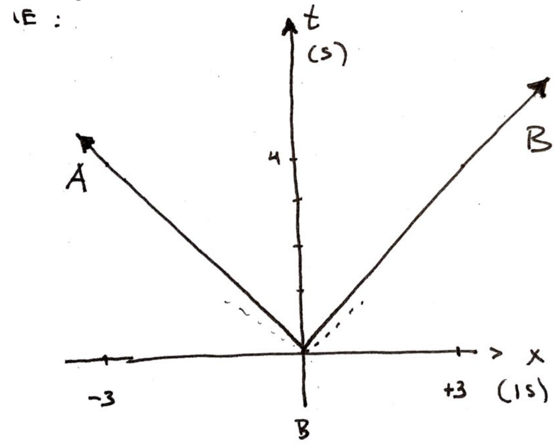
(7) *Spacetime diagrams* Let Harold have clock time t' and Maud have time t.
(a) Here's the spacetime diagram with parts (a) and (b)



(b) As shown
(c) Harold travels 1 light second (3×10^8) m in 2 seconds. Maud can expect a reply at $t = 3$ s or later since she sends the signal at 1 s, it takes 1 s to arrive at the rocket at 1 light second, and then 1s to return.

(8) Marcus-Ovid-Cattallus

(a) Here's a space-time diagram in Ovid's frame:



(b) This separation grows as the regular sum of the two speeds, so 1.5 c. This is consistent with SR since no object is moving as fast as this. No observer moves faster than c, as measured by another observer.

(9) The Hafele-Keating experiment

(a) My calculator returns "1".

(b) Using the very handy approximation

$$(1-x)^a \approx 1 - ax$$

for γ gives

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \simeq 1 + \frac{1}{2}\frac{v^2}{c^2}$$

with

$$\frac{1}{2}\frac{v^2}{c^2} \simeq 3.5 \times 10^{-13}$$

which is really small!

(c) If the flight took 14 hours then the difference in the clock readings due to special relativistic effects would be

$$t' - t = (\gamma - 1)t \simeq \frac{1}{2}\frac{v^2}{c^2}t$$

so that numerically

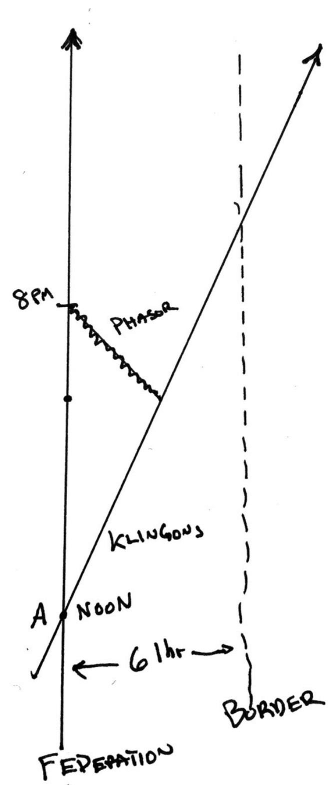
$$t' - t \simeq (3.5 \times 10^{-13})(14 \text{ hours})(3600 \text{ s/hour}) \simeq 1.8 \times 10^{-8} \text{ s}$$

or about 18 ns. This is also tiny - but measurable using atomic clocks!

Hafele and Keating had to account for general relativistic effects as well. With these effects, they found agreement between their experiment and the theoretical predictions.

(10) (2 pts.) A Federation cruiser, a Klingon battleship, and the border

(a) Here's a sketch of the spacetime diagram of the history in the cruiser's frame, with spaceships, border, and phaser.



(b) The Klingon ship passes the border, according to the Federation cruiser at

$$t = \frac{d}{v} = \frac{6 \text{ hrs } c}{3/5c} = 10 \text{ hrs},$$

past noon, or 10 PM. No SR here, just speed and distance in the Federation frame.

(c) The phasor arrived at 8 PM. Since the phasor is light, its worldline travels on a 45. This line intersects the Klingon's worldline at the launch event. These two conditions are satisfied if the phasor is launched at 5 PM from a distance of 3 lhrs, just where the Kilingon battleship is at 5 PM traveling at v = 3/5.

More formally, in the Federation frame the phasor was fired by the Klingon battleship at some distance D and some time t. Then, since the battleship was moving at 3/5c,

$$D = vt = \frac{3}{5}c\,t.$$

The light traveled the same distance from that time to 8

$$D = c(8 - t)$$

so that

$$\frac{3}{5}c\,t = c(8 - t) \text{ or } \left(\frac{3}{5} + 1\right) t = \frac{8}{5}t = 8$$

and $t = 5$ PM.

To summarize the history of events in the Federation cruiser's frame

- Noon - ships pass
- 5 PM Klingon battleship fires phasor in Federation territory
- 8 PM Phasor hits cruiser

- 10 PM Klingon ship passes into Klingon territory

(d) In the Klingon frame the ships passed at 12 noon. We can obtain the clock time when the phasor was fired using the redshift factor K. For this speed $K = 2$. So from the spacetime diagram,
$$T' = KT \text{ or } 8 = 2T \implies T = 4$$
So the phasor is fired at 4 PM in the Klingon's frame.
The factor γ for 3/5 c is 5/4. The Klingon ship measures a contracted distance to the border,
$$d = \frac{6}{\gamma} = 6\frac{4}{5} = \frac{24}{5}$$
or 4 4/5 light hours away. So the ship reaches the border at time t_c
$$\frac{24}{5} = \frac{3}{5}t_c \text{ so } t_c = 8 \text{ PM.}$$
We can use time dilation to find the time the phasor hits the Federation cruiser. Since the phasor arrives at 8 PM in the Federation's frame, which is moving relative to the Klingons, the time is
$$t_h = \gamma 8 = \frac{5}{4}8 = 10$$
and so 10 PM.[1] Plenty of SR in this part!
To summarize the history of events in the Klingons' frame
- Noon - ships pass
- 4 PM Klingon fires phasor
- 8 PM Klingon battleship ship passes into Klingon territory
- 10 PM Phasor hits cruiser

(e) As explained above, yes.

(f) The law is written with a preferred frame, which yields such messes. (And why is the border 'at rest' in the Federation frame anyway?) Instead the treaty could be re-worded to something like "it is illegal for a Klingon (Federation) ship in Federation (Klingon) territory to fire upon Federation (Klingon) property." Alternatively, it could be re-worded to fix the description of the border and hold in all frames (anticipating the upcoming interval).

[1] From the diagram it looks like there is some event between before 8 PM on the Federation worldline that is simultaneous with the Klingon ship crossing the border, in the Klingon frame. The slip in simultaneity between these two events in the Federation frame is then
$$\Delta t = \frac{vD}{c^2} = \left(\frac{3}{5}c\right)\left(\frac{6 \text{ hrs } c}{c^2}\right) = \frac{18}{5} \text{ hrs}$$
or 3 3/5 hours before 10 PM. This works out to $10 - 3.6 = 6.4$ hours or 6:24 PM. So by the Federation's clocks, the Klingon battleship passes into Klingon territory before the phasor hits the Federation cruiser, using the simultaneous events in the Klingons' frame.