

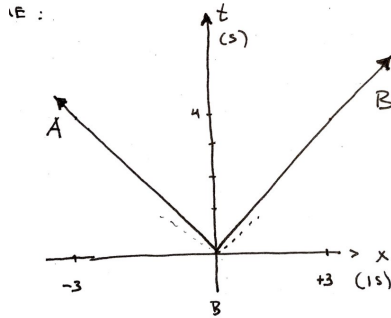
1. SOLUTIONS:

(1) Marcus, Ovid, and Cattallus

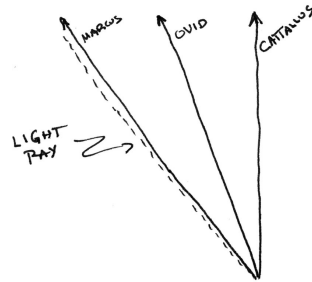
(a) This is a velocity addition computation

$$v' = \frac{\frac{3}{4}c + \frac{3}{4}c}{1 + 9/16} = \frac{24}{25}c \simeq 0.96c$$

(b) This separation grows as the regular sum of the two speeds, so $1.5c$. so you can see from the spacetime diagram in Ovid's frame



(c) Here's the spacetime diagram



The key part of this to to make sure that Marcus' worldline is not past the light ray, on the 45 degree diagonal line.

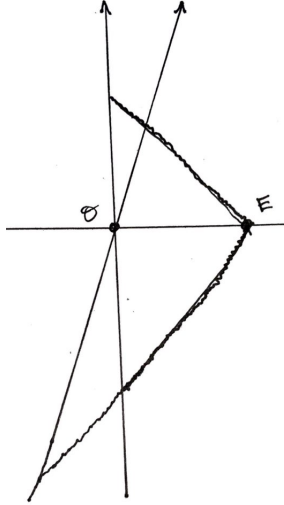
(d) Yes, it is consistent with SR since no object is moving as fast as $1.5c$ relative to another observer.

(2) In example 2 there are a bunch of marked lines about at the same wavelength as the H_α line so it is a little hard to read. But I estimate that the line is about 7200 angstroms or 720 nm ($= 10^{-9}$ m). This means that

$$z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} = \frac{720 - 656}{656} \simeq 0.098$$

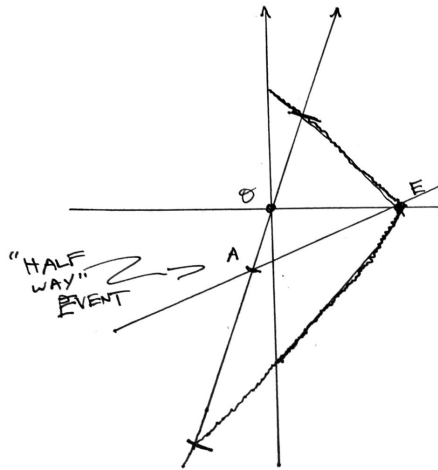
or about 0.1.

- (3) It's a funny thing about relativity. First, what are simultaneous events? An event on our worldline, say $\mathcal{O}$, simultaneous with a distant event E is the event that is at the “half-way time” between emission and reception of light sent to the distant event. For an observer at rest in the frame it looks like this



Event $\mathcal{O}$ is half way between when the light passes the observer, reflects off event E and returns. So $\mathcal{O}$ and E are simultaneous.

Notice the role of light. In any other frame light still travels at the same 3×10^8 m/s. So in any other frame this “half-way” event definition will create a slip in simultaneity. This is easiest to see in the diagram.



Now, due to the relative motion, we can see that the “half-way” event A is certainly not when (or where) O is.

(4) A *really* fast car:

- (a) Light changes color this way. When objects are receding the colors are shifted toward the red part of the spectrum and longer wavelengths. When objects are approaching, the light is shifted toward the blue. Visible light is ordered as Red Orange Yellow Green Blue Indigo Violet. So the red light was blueshifted to green. Using the relation among wavelengths, $\lambda' = K\lambda$, we have

$$K = \frac{\lambda'}{\lambda} = \frac{550}{650} \simeq 0.85$$

- (b) Using the relation between K and the speed

$$v = \frac{K^2 - 1}{K^2 + 1} c. \text{ and so } v \simeq -0.16c \simeq -5 \times 10^7 \text{ m/s} \simeq -1 \times 10^4 \text{ mph}$$

or 10,000 mph toward the light. That is fast! (Thankfully we are ignoring the dramatic effects that the atmosphere would have on the car.)

- (c) For 0.01 dollar for every mile per hour over 45 mph, this is expensive, about 1 million dollars. Might need to trade in the car!

(5) It is the use of ‘see’ that is incorrect. As we now know ‘seeing’ is a whole different set of effects. Instead, one way to correct the time dilation statement is, “The time $\Delta t'$ between two ticks on a clock measured by S in the ‘moving’ frame S' is larger than the time Δt between these ticks *in* the rest frame of the clock.”

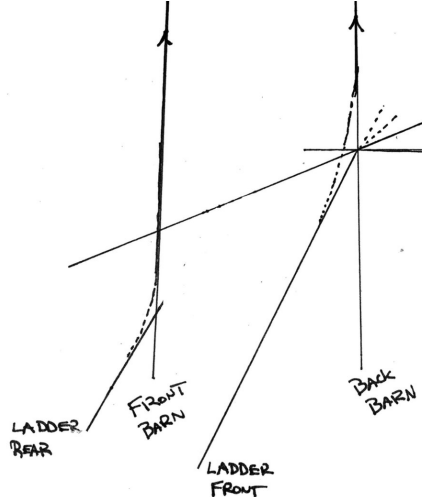
(6) The ‘ladder in barn’ situation with the speed of the runner at $0.866c$ and both proper lengths are 5 m.

(a)

$$\gamma = \frac{1}{\sqrt{1 - (0.866)^2}} \simeq 2$$

- (b) Use length contraction to obtain $L' = L/\gamma = 5/2 = 2.5$ m for the length of the ladder in the barn’s frame.

- (c) Symmetrically, the length of the barn in the ladder's frame is $5/2 = 2.5$ m
- (d) If the runner arranges to come to a stop when the front of the ladder reaches the back of the barn in the proper frame of the runner then the ladder does not fit in the barn. When the ladder starts to slow down only one half of its length is in the barn. As it slows the barn expands and, in the end, both the ladder and barn have the same length.
- (e) It may be easiest to see in a spacetime diagram. In the barn's frame,



The dashed lines show the ladder during its deceleration. The history in the barn's frame is: The front of the ladder arrives and then the rear of the ladder nears the front of the barn as the ladder slows. As it comes to rest both barn and ladder have the same length.

To construct this diagram I first started with the (contracted) ladder passing straight through. Then I drew the surface of simultaneity for the event when the ladder arrives at the back of the barn. This is when, in the ladder's frame, the runner and ladder arrange to stop. I smoothed out this stop - the 'arranging' - with dashed lines for the front and back of the ladder. When it stops, the ladder is the same length as the barn.

- (7) First in the ground frame. The trains accelerate identically and so remain the same distance apart. But as the cable (and trains) move faster they shrink. The cable snaps at 1.01 times its length so

$$1.01 = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \text{ or } \frac{v^2}{c^2} = 1 - \frac{1}{1.01^2} \text{ and } v \simeq 0.14c.$$

(Equivalently, one can start from $0.99 = 1/\gamma$.)

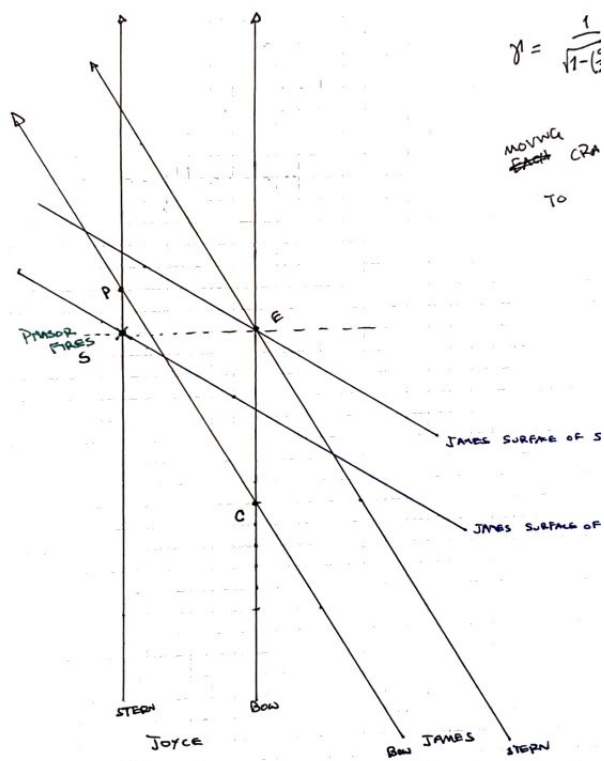
From the point of view of the train car in front, looking at a surface of simultaneity we can see that after the front car has already started accelerating the rear car is still not started yet! From the point of view of an observer in the train we can see that the front train started too early.

- (8) James and Joyce: At $v = 3/5c$, $\gamma = 5/4$.

(a) James' craft is contracted to

$$\frac{L}{\gamma} = \frac{4 \cdot 300}{5} \simeq 240 \text{ m}$$

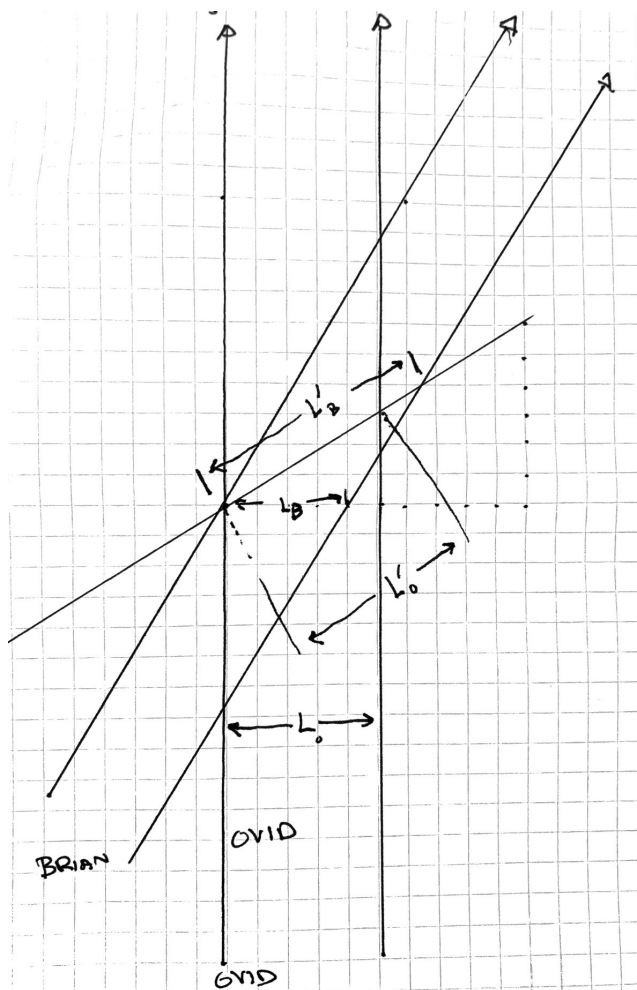
(b) Here's the spacetime diagram



(Sorry for the extra notes in the upper right.)

- (c) In James' frame: Bows pass, Joyce fires phasor and misses as intended, James' bow passes Joyce's stern, James' stern passes Joyce's bow, James' stern passes Joyce's stern.
- (d) Yes, it misses.

- (9) For these two equal-proper-length whales $\gamma = 5/4$. On Ovid's frame the contracted length of Brian is L/γ . I chose $L = 5$ m for the length for easy drawing. Here's the spacetime diagram for Ovid:



This diagram is to scale so it accurately reflects events, lengths, etc. L_B is the length of Brian in Ovid's frame, 4 m. L_O is Ovid's length of 5 m. L'_O is the length of Ovid in Brian's frame. L'_B is the length of Brian in Brian's frame. From the diagram it is easy to see that $L_B < L_O$ and $L'_O < L'_B$. These are both true.

- (10) A near-light speed trip:

- (a) The relation of proper time to the "lab" or Earth's frame is γ . Since this craft travels nearly the speed of light, the Earth time for the trip is 287 years or 2.51×10^6 hrs. so

$$\gamma = \frac{t_E}{t_P} \simeq \frac{2.51 \times 10^6}{52} \simeq 4.8 \times 10^4.$$

That's big!

- (b) Since

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

then

$$\frac{c-v}{c} = 1 - \sqrt{1 - \frac{1}{\gamma^2}} \simeq 1 - \left(1 - \frac{1}{2\gamma^2}\right) = \frac{1}{2\gamma^2} \simeq 2.1 \times 10^{-10}.$$

The ship's speed differs from the speed of light by 2 parts in 10^{10} . It is fast! In that middle approximation I used the handy relation $(1+x)^n \simeq 1+nx$.