Spacetime and the Quantum: Guide 5 Solutions

Fall 2025 v1.0

All questions were worth 1 point.

(1) 4.1 A sketch shows that the projections are (a) +3, (b) -3, (c) 0, (d) 0, (e) 0, (f) $3\sqrt{2}/2 \approx 2.121$. All results are in inches.

(2) 4.2 All the atoms exiting the "-" exit are in the $m_z = -m_B$ state. So if these atoms were sent into another z-Stern-Gerlach apparatus, they would all exit the "-" exit.

(3) 4.3 If the probability of an event is 1 then it certainly will happen. As (former 135 student) Oliver writes, "The claim and the experiment can be reconciled by recognizing that the probability of the atom leaving through the + exit is 1, which is the total probability for the atom, so all the atoms leave through the + exit."

(4) 4.4 As experiment 4.3 shows the only viable statement is (2). The atom does not have m_z once it leaves the B analyzer. It forgets that it had a m_z value.

(5) 4.5 By experiment 4.1 they would all leave the "-" exit in the first case. In the second case, as experiment 4.3 shows, each outcome would be equally likely. Half would leave the $m_z = +m_B$ exit and half would leave the $m_z = +m_B$ exit. The atoms would no longer have a value of m_z.

(6) 4.7 Since the results show that it is only the *relative* angle that is important, there would be no change.

(7) 4.9 The probability is 1/2, as we can see from the chart we used in class for 90 degrees. The relative angle is the only relevant angle.

(8) 4.10 The atoms that enter analyzer B are in the $m_z = +m_B$ state. From the chart, the probability of exiting the + exit of B is 3/4. Those atoms, in the state $m_60 = +m_B$, pass on to the C analyzer. Since the axes of the B and C analyzers is 60 degrees apart, the probability of exiting the + exit of C is 3/4 and the probability of exiting the - exit of C is 1/4.

(9) 5.1 To have 4 dots we need two 1's and one 2. The list is $(2, 1, 1)$, $(1, 2, 1)$ and $(1, 1, 2)$. The total probability for the three possible combinations is

$$P = 3 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{72}$$

(10) 5.3 Hmm, how many combinations have a sum more than 4? (6,6) obviously, and (5,6) (6,5), (5,4)... gracious, there are a lot of these possibilities! The case for which the sum is 4 or less is a lot easier. This probability of that outcome, rolling (1,1), (2,2) (2.1), (3,1), and the other orders, is 6/36. Thus,

$$P = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$$

(11) 5.4 The probability of either outcome (H or T) is 1/2 so for the sequences (a-c) of 10 coin tosses, the probability is

$$P = \left(\frac{1}{2}\right)^{10} \approx 9.8 \times 10^{-4}$$

Since the order is not specified in part d, the tail could occur in any one of the tosses so we have

$$P = 10 \times \left(\frac{1}{2}\right)^{10} \approx 9.8 \times 10^{-3}$$

(12) This question asks for the total probability for all three orientations or settings. So (a) the probability to leave the + axis of axis **a** is 0 since the state is $m_z = -m_B$. The probability to leave the + exit of axis **b** is 1-1/4 = 3/4 (The probability of the + outcome given $m_z = +m_B$ and axis **b** is 1/4, from the chart.) The **c** axis at 240 degrees has the same probability. Hence the total probability is $\left(\frac{3}{4} + \frac{3}{4}\right)/3 = 1/2$. (b) and (c) Since all the axes are 120 degrees apart the outcomes are the same, by symmetry. So the probability is 1/2. Only the relative angle is important.

(13) 5.6
 (a) Since after drafting the summer-born (400) the military needs another 300, half of the winter-born (600) will be called up. So any individual will be called up with probability 1/2, if they are randomly selected.
 (b) By the same logic as in part a, after the winter-born are drafted the military needs another 100, so 100/400 = 1/4 as expected.
 (c) Well, we've done the two cases for one group being drafted first so we have, for winter-born

$$\frac{1}{2} \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{3}{4},$$

 and for summer-born

$$\frac{1}{2} \times \frac{1}{4} + 1 \times \frac{1}{2} = \frac{5}{8}.$$

 It all looks pretty lopsided - Ooops.

(14) Questions on plants
 (a) To survive the week the planet it either is watered properly (with probability $1 - .3 = .7$) and survives (with probability $1 - .2 = .8$), or it is sadly neglected (with probability 0.3) and still makes it (with probability $1 - .9 = .1$) so the total probability is

$$P(\text{survive!}) = 0.7 \cdot 0.8 + 0.3 \cdot 0.1 = 0.59$$

 or 59%.
 (b) If the plant dies then either our friend forgot to stop by (with probability 0.3) and the plant ended up dying (with probability 0.9 so a total probability of $0.3 \cdot 0.9$) or our friend remembered (with probability 0.7) and even so the plant died (with probability 0.2 so a total probability of $0.7 \cdot 0.2$). We are looking for the fraction of "our friend forgot" out of all "plant dies" scenarios. Thus,

$$P(\text{oops-forgot}) = \frac{0.3 \cdot 0.9}{0.3 \cdot 0.9 + 0.3 \cdot 0.2} = \frac{27}{41} \simeq 0.658$$

 or about 66%.