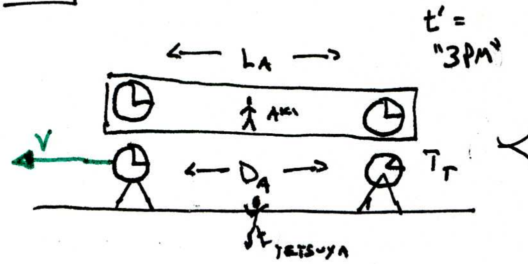
LENGTH CONTRACTION V2

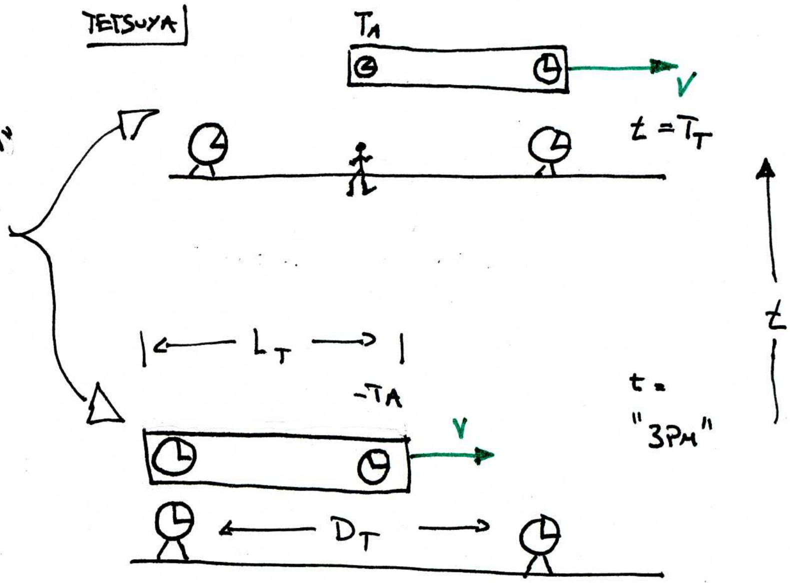
FALL 2008

SPATIAL SNAPSHOTS

AKI



TETSUYA

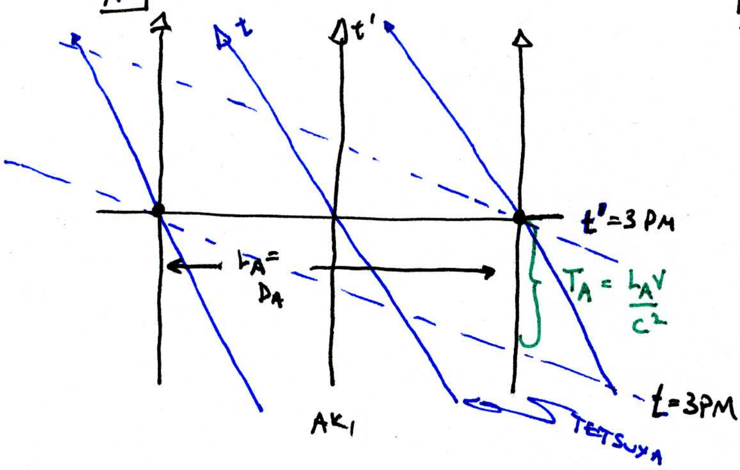


BY CHOICE, $\boxed{L_A = D_A}$ (1)

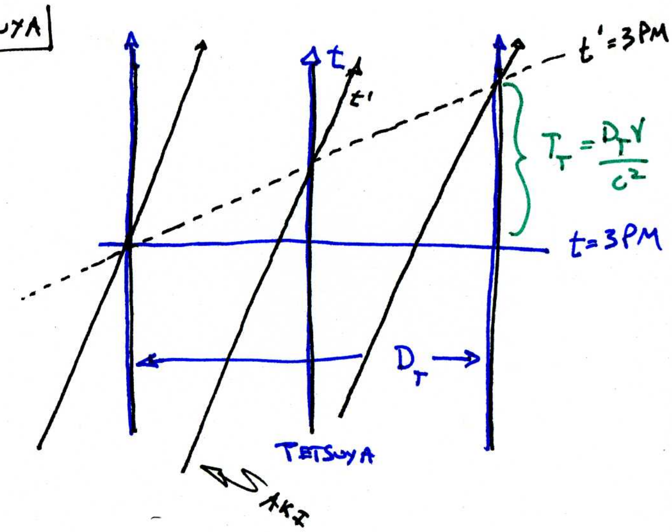
(WITHOUT LOSS OF GENERALITY)

SPACE-TIME DIAGRAMS

AKI



TETSUYA



BY THE RELATIVITY PRINCIPLE

$$\boxed{\frac{L_T}{L_A} = \frac{D_A}{D_T}} \qquad (2)$$

FROM THE SPACE-TIME DIAGRAMS (AND THE ABSTRACT "RULE") $\boxed{T_T = \frac{D_T V}{c^2}}$ (3) AND $\boxed{T_A = \frac{L_A V}{c^2}}$ (4)

FROM THE TETSUYA SPATIAL SNAPSHOTS, $\boxed{D_T = L_T + V T_T}$ (5)

NOW ALGEBRA! (3) AND (4) GIVE

$$\frac{T_A}{T_T} = \frac{\frac{L_A V}{c^2}}{\frac{D_T V}{c^2}} = \frac{L_A}{D_T} \overset{\text{FROM (1)}}{=} \frac{D_A}{D_T} \overset{\text{FROM (2)}}{=} \frac{L_T}{L_A} = \frac{1}{\gamma} \qquad (6)$$

[WHAT WE CALLED "m" FOR MYSTERY IN CLASS]

FROM (5) AND (3) $D_T = L_T + \frac{D_T V^2}{c^2} \Rightarrow L_T = D_T\left(1 - \frac{V^2}{c^2}\right)$ (7)

WHILE FROM (6)

$$L_T = \frac{1}{\gamma} L_A \overset{\text{FROM (1)}}{=} \frac{1}{\gamma} D_A \overset{\text{FROM (6)}}{=} \frac{1}{\gamma^2} D_T \qquad (8)$$

THEREFORE, COMBINING (7) AND (8),

$$L_T = \frac{1}{\gamma^2} D_T = D_T \left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \quad \frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} \quad \Rightarrow \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

SO, FROM (6) AGAIN

$L_T = \frac{1}{\gamma} L_A$, IN WORDS "MEASURED IN A MOVING FRAME, AN OBJECT'S PROPER LENGTH L_A IS CONTRACTED BY A FACTOR OF $1/\gamma$"

– ADAPTED FROM MERMIN, IT'S ABOUT TIME PP. 58–69