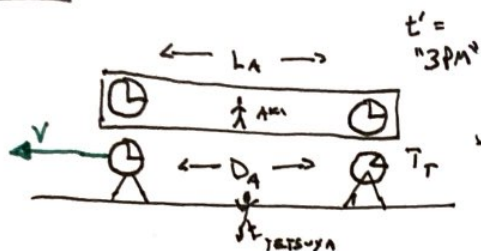


LENGTH CONTRACTION V2

SPATIAL SNAPSHOTS

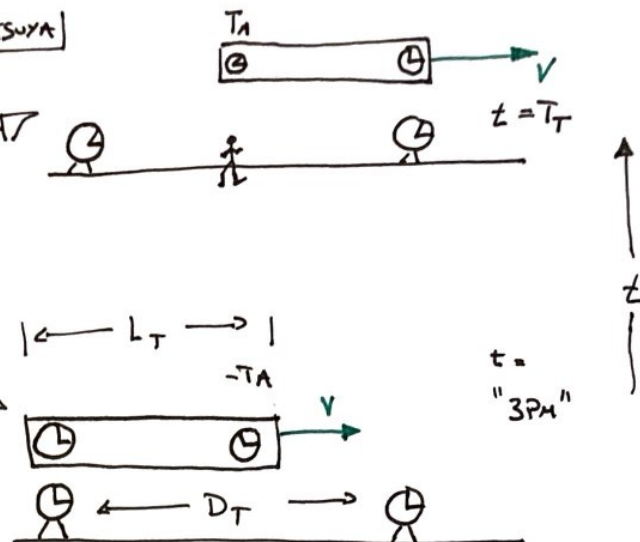
AKI



By choice, $L_A = D_A$ (1)

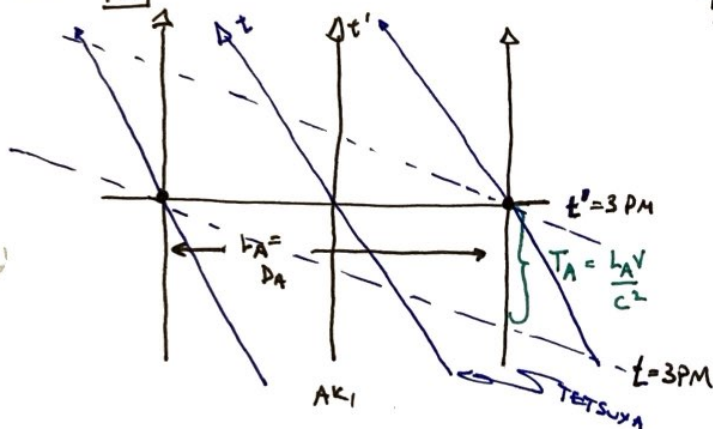
(WITHOUT LOSS OF GENERALITY)

TETSUYA

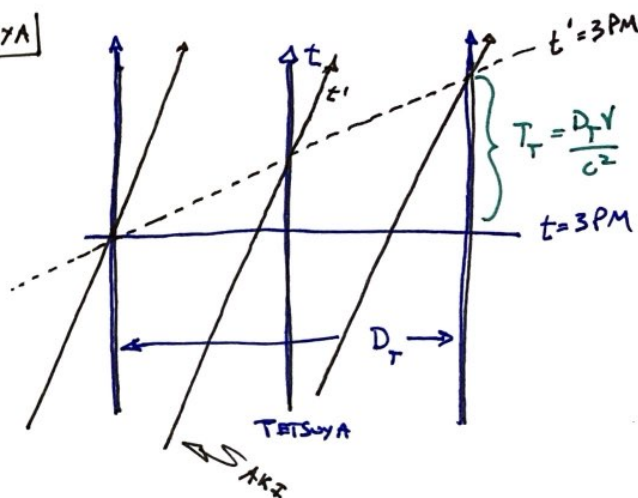


SPACE-TIME DIAGRAM

AKI



TETSUYA



BY THE RELATIVITY PRINCIPLE

$$\frac{L_T}{L_A} = \frac{D_A}{D_T} \quad (2)$$

FROM THE SPACE-TIME DIAGRAM
(AND THE ABSTRACT "RULE")

$$T_T = \frac{D_T v}{c^2} \quad (3)$$

$$T_A = \frac{L_A v}{c^2} \quad (4)$$

FROM THE TETSUYA SPATIAL SNAPSHOTS,

$$D_T = L_T + v T_T \quad (5)$$

NOW ALGEBRA! (3) AND (4) GIVE

$$\frac{T_A}{T_T} = \frac{\frac{L_A v}{c^2}}{\frac{D_T v}{c^2}} = \frac{L_A}{D_T} = \frac{D_A}{D_T} = \frac{L_T}{L_A} = \frac{1}{\gamma} \quad \left[\text{WHAT WE CALLED "M" FOR MYSTERY IN CLASS} \right] \quad (6)$$

FROM (5) AND (3) $D_T = L_T + \frac{D_T v^2}{c^2} \Rightarrow L_T = D_T \left(1 - \frac{v^2}{c^2} \right) \quad (7)$

WHILE FROM (6)

$$L_T = \frac{1}{\gamma} L_A \stackrel{\text{FROM (1)}}{=} \frac{1}{\gamma} D_A \stackrel{\text{FROM (6)}}{=} \frac{1}{\gamma^2} D_T \quad (8)$$

THEREFORE, COMBINING (7) AND (8),

$$L_T = \frac{1}{\gamma^2} D_T = D_T \left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} \Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

SO, FROM (6) AGAIN

$$L_T = \frac{1}{\gamma} L_A$$

, IN WORDS "MEASURED IN A MOVING FRAME,
AN OBJECT'S PROPER LENGTH
 L_A IS CONTRACTED BY A
FACTOR OF $1/\gamma$ "

- ADAPTED FROM MERMIN, IT'S ABOUT TIME PP. 58-69