PHYS 135: Spacetime Quiz Combined Questions

Fall 2025 v1.0

1. Instructions

This quiz is under the auspices of the Hamilton Honor Code.

- You may use a calculator.
- Please ask questions, particularly when the statement of the question is confusing.
- Other than me, you may not consult with any person, text or resource via any means.
- You may not share this quiz
- You have 75 minutes to complete your solutions.
- There are 4 questions and a total of 65 possible points.
- May the light be with you!

2. Some of our favorite expressions

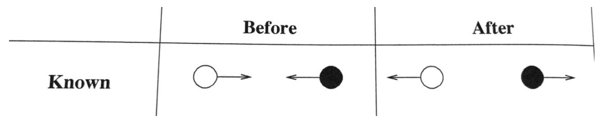
Please use the speed of light $c = 3.0 \times 10^8$ m/s.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ and } K = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \tag{1}$$

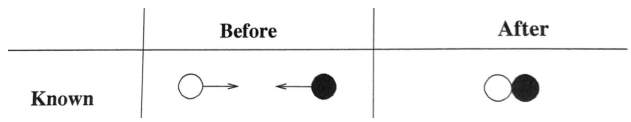
- "Moving objects shrink" or length contraction. A moving object's proper length L_p contracts as
$$L = \frac{L_p}{\gamma} \tag{2}$$
- "Moving clocks run slow" or time dilation. A moving clock's time t' runs slow as
$$t = \gamma t' \tag{3}$$
- "Slip in simultaneity". The time in a moving frame between simultaneous events in another frame is
$$T = \frac{vD}{c^2} \tag{4}$$
The details: If events E_1 and E_2 are simultaneous in one frame then in a frame moving with speed v in the direction from E_1 to E_2, the event E_2 occurs earlier than E_1 by the time interval Dv/c^2, where D is the distance between the events in the second frame.
- "Velocity addition is modified". An object moves at u in a frame. In another frame moving at v with respect to this frame, the object moves at w given by
$$w = \frac{v + u}{1 + uv/c^2} \tag{5}$$
- "Light from moving sources changes color". For observers with relative velocity v
$$K = \frac{T'}{T} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} = \gamma + \sqrt{\gamma^2 - 1} \tag{6}$$
where T' is in the frame that receives the light, T is in the frame that emits light. The wavelengths are related by
$$\lambda' = K\lambda$$
For receding observers $v > 0$, for approaching observers $v < 0$. The speed in terms of K is given by
$$\frac{v}{c} = \frac{K^2 - 1}{K^2 + 1} = \sqrt{1 - \frac{1}{\gamma^2}} \tag{7}$$

The **known situations** are:

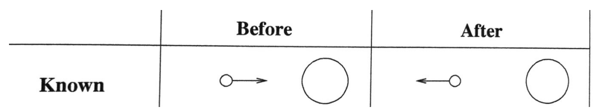
(1) For identical objects undergoing elastic collisions



(2) For identical objects undergoing sticky (completely in-elastic) collisions:



(3) For the "big ball - small ball" collision:



3. Questions: Please include explanations, calculations, and/or diagrams so the logic of your solution is clear

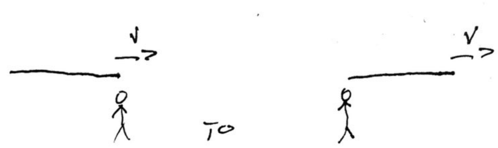
(1) (15 pts.) Two, equal mass balls collide elastically. Before the collision, a green ball moves to the right at a velocity of 4 ms^{-1} and a blue ball moves left at 2 ms^{-1}.

(a) Using the switching frames method, find the final velocities of the green and blue balls in the original frame. In your answer specify which frame is the known situation.

(b) Sketch a spacetime diagram of the collision in the original frame. Since the velocities are small please use units like seconds and meters rather than, for instance, seconds and light seconds.

(2) (10 pts.) Two identical, "sticky" pucks collide. What happens if the one on the left, moving at 4 ms^{-1} to the right, collides with a stationary puck on the right? Please use the "switching frames method" and present your solution.

(3) (20 pts.) A stick with proper length $L = 2$ m moves by you at speed $v = 3/5c$. Here's a sketch



(a) What is the time interval between when the front of the stick passes you and when the back passes you?

(b) Sketch a spacetime diagram of the events in your frame. Include worldlines of the front and back of the stick, and you. Show the time interval you found in part (a).

(c) In the stick's frame what is the time interval between these passing events ? Add this time to your spacetime diagram.

(4) (20 pts.) A train is 30 m long in its proper frame and consists of an engine and two cars, each of equal length. It moves so fast that it is only 6 m long in the Earth's reference frame. It speeds through a "falling rock zone" and, as bad luck would have it, a huge rock (15 m long in its own frame) tumbles down the mountainside and completely crushes the 6 m long train. Or does it?

(a) In the reference frame of the train, how long is the rock?

(b) What size hole would this rock make in the roof of one of the train cars, in the train's frame?

(c) Is the train is either completely squashed or is one car damaged but not destroyed? Which analysis is correct? Discuss the situation. Feel free to draw a spacetime diagram and make assumptions about events (if you do, be sure to state them).

(5) (20 pts.) A Super Duper exploratory spacecraft departs from a space port near Earth. The spacecraft travels toward the red dwarf star Ross 154, about 10 lyr away (as determined in Earth's frame), at $\frac{4}{5}c = 0.8c$. At the same time, in Earth's frame, an older Super exploratory spacecraft passes Ross 154 traveling at $\frac{5}{8}c = 0.625c$ on the way back to Earth.

(a) Sketch a spacetime diagram of Earth, the star, and the two spacecraft.

(b) What is the speed of the Super spacecraft, according to the Super Duper spacecraft?

(c) At what rate are the two spacecraft approaching in Earth's frame? Comment on your result.

(d) Again in Earth's frame, when do the spacecraft pass each other?

(e) When do the ships pass, according to the clocks on the Super Duper spacecraft?

(f) What is the distance between Earth and the star Ross 154 in the Super Duper spacecraft frame?

(g) Consider the following argument: "According to observers on the Super Duper spacecraft, the initial distance to the Super spacecraft is given by your answer to part 5f. The time required to meet the Super spacecraft is given by that distance divided by the speed, which you found in part 5b." This, however, does not give the right answer, the one you found in part 5e. Why?

Hints: For the Super Duper spacecraft $\gamma = \frac{5}{3}$. For the Super spacecraft $\gamma \approx 1.6$. If you use the spacetime diagram to answer any of the following questions, make sure that you make this to scale.

(6) (15 pts.) The author of the book we will use for the "quantum" portion of this course recently wrote an article on special relativity. In the paper he claims that

> "In one frame two events are simultaneous and separated by a distance Δx. In a frame moving relative to the first at velocity v, the two events are separated in time by
>
> $$\frac{v\Delta x/c^2}{\sqrt{1-(v/c)^2}}." \tag{8}$$

Compare this to equation (4). Is there a typo in equation (8)? If so, how would you correct it. If the claim is correct, explain why the two expressions differ.