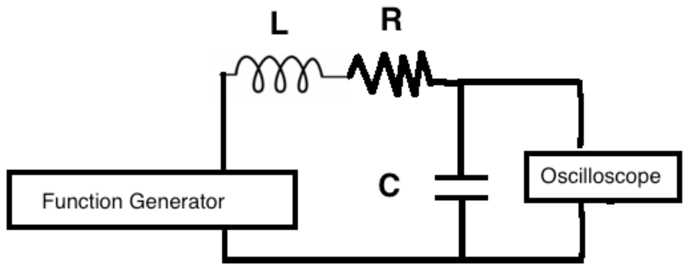
Resonance in Circuits

Purpose:

- To map out the analogy between mechanical and electronic resonant systems
- To discover how relative phase depends on driving frequency
- To gain experience setting up circuits

Apparatus: Oscilloscope, function generator, 10 nF capacitor, 10 mH inductor, 220Ω resistor, BNC cables, banana cables



Introduction:

As you recall we change the driving frequency of a damped, driven, harmonic oscillator the amplitude of motion and the relative phase between the driving force and the motion change. As you saw in Lab 4, the amplitude reaches a peak at the resonant amplitude. In this lab we explore the relative phase in an electronic analog of the damped, driven, harmonic oscillator.

LRC Circuit Introduction:

An LRC (or RLC or LCR) circuit is made of three electrical components, an inductor ("L"), a capacitor ("C"), and a resistor ("R"). Don't worry about the electric and magnetic properties of these components. We can use the following (exact) analogy with a mechanical system:

Mechanical		Electrical	
x	displacement	q	charge
dx/dt	velocity	$I=dq/dt$	current
m	mass	L	inductance
b	damping	R	resistance
k	spring constant	$1/C$	1 over capacitance
F_0	amplitude of force	V_o	amplitude of voltage

The inductor, capacitor, and resistor are connected one after the other - in "series" as shown by the diagram above. The order of the components doesn't matter. As you can see from the above table, LRC circuits are resonant systems; there is a special resonant frequency (ω_R) such that the amplitude of oscillation reaches a maximum and the relative phase reaches $\pi/2$. Assuming as we would for the mechanical system that the driver (the voltage of the generator) has a fixed amplitude V_o that is independent of the generator's frequency $f = \omega/2\pi$, then the amplitude of the charge that sloshes back and forth in the circuit is

$$q(\omega) = \frac{V_0/L}{\sqrt{\left(\omega^2 - \frac{1}{LC}\right)^2 + (\omega R)^2}}$$

This should look a bit familiar. The amplitude of a damped driven oscillator is given by

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + 4\omega^2\beta^2}}$$

The relative phase is

$$\phi = \arctan\left(\frac{2\beta\omega}{\omega_o^2 - \omega^2}\right)$$

where

$$\beta = \frac{b}{2m} = \frac{R}{2L}$$

In many practical situations, the resistance R is not only in a resistor but also "hidden" in the inductor and other components. In our case, the resistor has a value of 220 Ω.

Prelab: Read this lab
Develop a playful mind

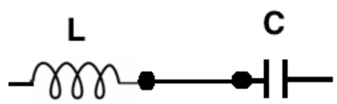
Mass on a Spring:
In Lab 4 on resonance, we explored the motion of a mass on a spring with magnetic damping while it was driven by a speaker. As the driving frequency approached the resonant frequency we noticed that the small driving amplitude of the speaker resulted in large amplitude motion of the mass.

An electric circuit shows the same behavior as a mass on a spring. However, instead of using position, velocity and acceleration to describe the motion, we use charge, current, and voltage. In circuits, the analogous physical parameter to position is charge. In the lab, we monitor the voltage across the capacitor which is related to the charge via $\Delta V_C = q/C$.

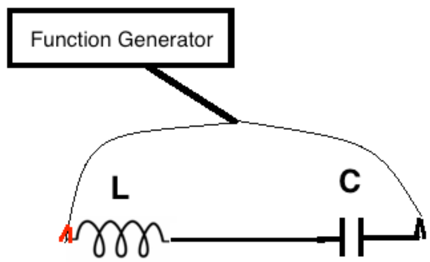
Building a LC Circuit to find the resonant frequency:
Before building the LRC circuit, we must first find the natural frequency of the circuit. While it is possible to calculate the resonant frequency of the circuit using $\omega_0 = 1/\sqrt{LC}$, this will likely yield an incorrect result. Unfortunately, the value of the inductance has an uncertainty of around 10%, and the uncertainty of the capacitance can be as large as 30%! Instead, we are going to determine our natural frequency experimentally similar to how we determined it for our mass on a spring system.

When we were looking at the mass on a spring system, we found the natural frequency by removing the magnetic damping, starting the mass moving with a large amplitude, and watching the motion with a motion detector. Then we fit the resulting sine wave to determine the natural frequency of the system. For the LRC circuit, we will do something similar, removing the resistor to reduce damping. We will start the charge moving by hitting the circuit with a square wave, and track the motion with an oscilloscope.

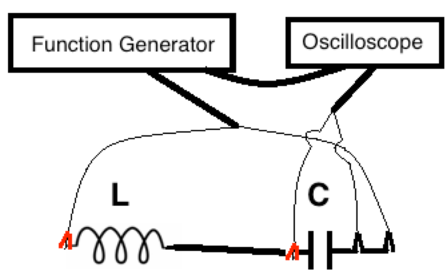
Let's start by building the circuit:
1. Connect the 10 mH inductor, and the 10 nF capacitor together such that the circuit makes one continuous path through all of the elements one after the other. Then these components are connected "in series" as in the diagram below. The dots represent connection points and the lines between them are wires. In future diagrams the dots will be left out.



2. Now connect the function generator in such a way that the black clip connects to the capacitor and the red clip connects to the inductor. This will drive this circuit with the square wave voltage, so be sure to select the square wave setting.



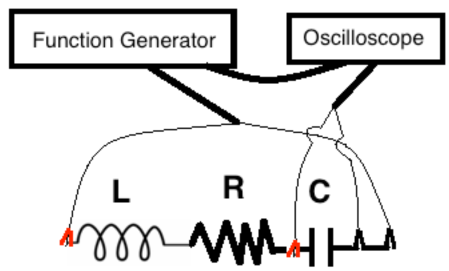
3. We use the oscilloscope to measure both the voltage from the function generator as well as the voltage from the circuit. Connect one channel of the oscilloscope directly to the function generator to measure the input voltage (V_{in}). Then connect the next channel of the oscilloscope to both sides of the capacitor to measure the voltage out from the circuit (V_{out}). Make sure to connect the red and black of the leads as described by the diagram below.



4. When you think your circuit is ready, turn on all of the devices and begin looking for the signals on the oscilloscope. Start by changing the frequency of the square wave pulse to around 10 Hz, and triggering on the signal generator's rising square wave. Once you have found the square wave from the signal generator, examine the signal across the capacitor. You will likely need to zoom in on the signal to see the oscillations of the circuit after the big pulse. Moving to the measure function should allow the oscilloscope to measure the frequency of the resulting wave giving you the natural frequency of the circuit. Write the natural frequency down in standard form!

Building the LRC Circuit and Exploring Relative Phase:
Now we will examine an LRC series circuit and the phase relationship. Much like when we explored the resonance curve for the mass on a spring, we will be using a damped, driven system. In this case, instead of driving with a speaker, we will be driving the system from a sine wave produced by the signal generator. Instead of magnetic damping we will be using the resistor to produce damping in the system.

1. Create the following LRC circuit:



2. When your circuit is ready, look for the signals on the oscilloscope. You can start by finding and triggering on the signal generator's sine wave.

3. Once you have found the sine wave from the signal generator, set the driving frequency of the signal generator near the natural frequency of the circuit. As you approach resonance, you should start to see a signal from voltage across the capacitor. If you do not, find your instructor for help.

4. Explore the frequencies around the resonant frequency of your system. Watch the amplitude and phase of the voltage across the capacitor changes as you change the frequency. You are observing the resonance of your LRC circuit. Neat! You've built your very own electric resonant system!

5. In an LRC circuit, we expect that on resonance, the phase of the current (which is proportional to V_{in}) should be in phase with the voltage across the resistor, 90° behind the voltage across the inductor, and 90° ahead of the voltage across the capacitor (V_{out}). Let's put this to the test!

6. Devise a method to determine the phase difference between the voltage from the signal generator and the voltage across the capacitor.

7. Take data on the phase around the resonant frequency. In your favorite spreadsheet, build a plot of your phase angle vs. driving frequency that will allow for more data to be added later.

8. Propagate error in your measurements and add vertical error bars to data on your graph.

9. Using the equation for phase and known values, create a theoretical curve over your data. Enter this so you can change the value for the resistance *R* and the graph will update the theoretical curve. Start by making a column of dummy values for frequency and finding the phase based on those frequencies. You should have a smooth theoretical line over your data if done correctly.

10. Add experimental data to fill in gaps and see if experimental data and theoretical prediction agree well.

11. Adjust the resistance value for a better fit. What is the change you had to make to the resistance of the circuit? Why do you think this is necessary? What is your uncertainty value?

12. Print out one copy of this graph for your lab notebook and one copy to turn in for the postlab.