Intro:

*Magnetic fields will be the topic this week. We’ll carefully study the field of a coil of wire and the torque on a current loop in a magnetic field. This is critical for the design of motors.*

Reading:
- Friday (April 16): HRW 29.1
- Monday: HRW 29.4, 28.1
- Wednesday: HRW 28.2-5, 28.6-8, and 29.5

Physics Topics:
- Biot-Savart field
- Lorentz force \( qv \times B \)
- Magnetic moment
- The \( I\ell \times B \) force

Math Topics:
- Cross products (review)
- The Lorentz force
- Current loops and magnetic torque

Problems: *Optional mid-term questions*

1. Equations and Derivations - choose one of the following:
   (a) Derive the wave equation for transverse waves on a string. Assume the string has linear mass density \( \mu \) and tension \( F_T \). Start with a careful diagram and include explanations of your steps. What is the wave speed?
   (b) Derive the wave equation for sound waves in a fluid. Assume a bulk modulus of \( B \) and a density \( \rho \). Start with a careful diagram and include explanations of your steps. What is the wave speed?

2. In 2014 scientists announced the discovery of an underground sea of liquid water on the moon Enceladus, which orbits Saturn. They found the sea by carefully mapping the gravitational field of the moon. They tracked the Cassini spacecraft using radio waves. Kenneth Chang, writing in the New York Times, described the method, “As the pull of Enceladus’s gravity sped and then slowed the spacecraft, the frequency of the radio signal shifted, just as the pitch of a train whistle rises and falls as it passes by a listener.” Radio signals travel at \( c = 3 \times 10^8 \) m/s.
   (a) What is the name of the physical effect used to find the sea?
   (b) They had astonishing precision and were able to measure a change of velocity as small as \( \delta v = 0.09 \) mm/s. What relative frequency difference \( (\delta f/f) \) must they been able to measure?
   [If you are curious how this is possible, the experimenters used atomic clocks, which keep time within a part in \( 10^{14} \) to \( 10^{18} \).]

3. In the lab room there are some funny plastic tubes. If you whirl them around your head (please try it!) they make sounds.
(a) What is a possible explanation of this phenomenon?
(b) What is the length of one of the tubes?
(c) Estimate the lowest frequency produced by the whirling tube.
(d) Derive the relation for all of the frequencies likely produced by the tube.

(4) Fourier: Using diagrams, examples, words, cartoons, and/or equations explain how any function can be expressed as a Fourier series. Why is this a useful way of expressing functions and expressing signals (and viewing the world)?

(5) A point-like source of sound has a power of 1.2 µW. What is the intensity 3.0 m away? What is the sound level there in decibels?

(6) Organ sounds: Suppose that you have two organ pipes, each with one open and one closed end. One pipe has a length of 1.26 ± 0.02 m and the other has a length of 1.22 ± 0.02 m.
   (a) If you played them simultaneously in their second harmonic describe in words what you would hear. Include a sketch of the harmonic.
   (b) Find the frequency of the sound that you would hear. Please include the uncertainty.
   (c) Find the frequency of variation in the volume (the “wha-wha”’s) that you would hear. Please include the uncertainty.
   For this question please assume \( v = 343 \pm 2 \) m/s.

(7) Two sinusoidal waves of the same frequency travel in the same direction along a string. If \( D_{m1} = 2.2 \, \mu m, D_{m2} = 4.2 \, \mu m, \) and the relative phase is \( \phi = \pi/6, \) what is the resulting wave’s amplitude and phase?

(8) A source \( S \) of sound waves and microphone \( M \) in water are a distance \( s \) apart just below the surface of the ocean. The sound waves reach the mic either directly through the water or by reflecting off a cold, denser water layer in the ocean at a depth \( D. \) The wave and reflected wave are in phase when the layer is at this depth. If the layer gradually sinks, the phase difference between the two waves gradually shifts until they are exactly out of phase at a depth of \( d + D \) as shown. Find the wavelength of the sound \( \lambda \) in terms of \( d, D, \) and \( s. \)

Lab:
will instead be Mid-term II...

A look ahead. . .
Waves in the form of light are coming up, see Chapter 33.
Handy Relations

General:

\[ \sum \tau = I \alpha \quad I = \int r^2 dm \]

\[ F = -\frac{dU}{dx} \]

\[ \Delta P = \frac{F}{A} = -B \frac{\Delta V}{V} \]

\[ F_B = \rho g V \]

The Taylor series of a function \( f(x) \) around \( x = x_o \) is

\[ f(x) = f(x_o) + \frac{df}{dx} \bigg|_{x=x_o} (x - x_o) + \frac{1}{2!} \frac{d^2f}{dx^2} \bigg|_{x=x_o} (x - x_o)^2 + \frac{1}{3!} \frac{d^3f}{dx^3} \bigg|_{x=x_o} (x - x_o)^3 + \ldots \]

Handy approximation:

\[ (1 + x)^n \simeq 1 + nx \text{ for small } x \]

Oscillations:

\[ F = -kx \text{ and } T = \frac{2\pi}{\omega} \text{ and } \omega = 2\pi f \]

For spring-like SHM \( \omega_o = \sqrt{\frac{k}{m}} \). For a simple pendulum \( \omega_o = \sqrt{\frac{g}{L}} \).

\[ Q = \frac{m\omega_o}{b} \]

\[ E = \frac{1}{2}m\omega_o^2A^2 \]

A damped harmonic oscillator has the equation of motion

\[ \frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega_o^2 x = 0 \text{ with } \beta = \frac{b}{2m} \text{. The angular frequency is } \omega_d = \sqrt{\omega_o^2 - \beta^2} \]

For a driven system

\[ \frac{d^2x}{dt^2} + 2\beta \frac{dx}{dt} + \omega_o^2 x = \frac{F_o}{m} \cos(\omega t) \]

at late times \( x(t) = A(\omega) \cos[\omega t - \delta(\omega)] \) with

\[ A(\omega) = \frac{F_o/m}{\left(\frac{\omega_o^2 - \omega^2}{\omega_o^2} + \frac{4\beta^2\omega^2}{\omega_o^2}\right)^{1/2}} \text{ and } \delta(\omega) = \arctan \left( \frac{2\beta\omega}{\omega_o^2 - \omega^2} \right) \]

\[ k_{eff} = \frac{d^2U}{dx^2} \bigg|_{x=x_{eq}} \]

Waves:

For waves on a string:

\[ y(x, t) = y_m \sin(kx \pm \omega t) \]

\[ k = \frac{2\pi}{\lambda}, v = \frac{\omega}{k} \]

Standing wave condition for waves fixed at both ends

\[ L = \frac{n\lambda}{2}, n = 1, 2, 3, \ldots \]

\[ v = \lambda f \]

\[ f = \frac{nv}{2L}, n = 1, 2, 3, \ldots \text{ for only one fixed end it is } f = \frac{(2n-1)v}{4L}, n = 1, 2, 3, \ldots \]
Phase:

\[ \frac{\varphi}{2\pi} = \frac{\Delta L}{\lambda} \]

Phasors: For two waves, with amplitudes \( y_{m1} \) and \( y_{m2} \) and relative phase \( \varphi \), resulting length of the sum is

\[ r = |z| = \sqrt{y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos \varphi}. \]

The phase is given by

\[ \tan \alpha = \frac{y_{m2}\sin \varphi}{y_{m1} + y_{m2}\cos \varphi}. \]

Sound:
The phase speed of sound in air is about \( c_s = 343 \text{ m/s} \).

\[ P = 2\pi^2 \mu v f^2 A_o^2, \quad I = 2\pi^2 v \rho f^2 A^2, \quad I = \frac{P_o}{4\pi f^2} \]

\[ \beta = 10 \log \frac{I}{I_o} \text{ with } I_o = 10^{-12} \text{ W/m}^2 \]

\[ f_b = f_1 - f_2, \quad d \sin \theta = m\lambda \]

\[ f' = f \left( \frac{c_s \pm v_0}{c_s \mp v_s} \right) \]

Light:

\[ f' = f \sqrt{\frac{c \pm v}{c \mp v}} \]

**Uncertainty:** If you like you are welcome to add independent uncertainties in quadrature.

For addition and subtraction, add the uncertainties in quadrature,

\[ z = x + y \text{ or } z = x - y \text{ then } \delta z = \sqrt{\delta x^2 + \delta y^2} \]

For multiplication and division then add the relative uncertainties in quadrature,

\[ z = xy \text{ or } z = x/y \text{ then } \frac{\delta z}{z} = \sqrt{\left( \frac{\delta x}{x} \right)^2 + \left( \frac{\delta y}{y} \right)^2} \]

For a power, multiply the relative uncertainty by the power, i.e. if \( z = x^n \) then \( \frac{\delta z}{z} = n \frac{\delta x}{x} \).

In general for a calculated quantity \( q = q(x, \ldots, z) \) then

\[ \delta q = \sqrt{\left( \frac{\partial q}{\partial x} \delta x \right)^2 + \ldots + \left( \frac{\partial q}{\partial z} \delta z \right)^2} \]
Solution Hints (Your solutions should be more discursive!):

(1) See class notes and the text.

(2) (a) Doppler or frequency shift for light
    (b) \[ f' \simeq f \left(1 + \frac{v}{c}\right) \]
        But the reflection gives twice the shift \( \frac{\delta f}{f} = \frac{2\delta v}{c} = 6 \times 10^{-13} \)

(3) (a) resonance
    (b) \( L = 76 \pm 1 \text{ cm} \)
    (c) 220 Hz
    (d) see notes - for open ends such as these
        \[ f = \frac{n \nu_s}{2L}, \quad n = 1, 2, 3, \ldots \]

(4) Fourier’s theorem says that ‘any’ function can be expressed as a sum of sines and cosines. Seeing which oscillatory functions are in the sum is a useful way to see which frequencies are in the function.

(5) \( I \simeq 1.06 \times 10^{-8} \text{ W/m}^2 \) and 40 dB

(6) (a) beats
    (b) \( 208 \pm 3 \text{ Hz} \)
    (c) \( 7 \pm 7 \text{ Hz} \)

(7) The magnitude is about 6.2 and the phase is about 0.34.

(8) \[ \lambda = 4\sqrt{(s/2)^2 + (D + d)^2 - 2s} \]