HYDROGEN SPECTRUM

By: Hamilton Physics Department

LEARNING OBJECTIVES

1. Describe the basic design of a spectrometer,
2. Determine the wavelengths of various visible lines of hydrogen, and
3. Mathematically model allowed electron transitions in hydrogen and compare to the measured values

What to expect during lab:

1. Seeing the glow of the hydrogen spectrum with your own eyes. (Room lights off!)
2. Exacting experimental work with high precision data-taking, measuring angles in degrees and minutes.
3. Creating a linear graph from a nonlinear equation.

PRE-LAB

We'll be using a Vernier scale this week. As an example, consider the figures below.

Figure 1: A measurement with a Vernier Scale

In figure 1, the main scale is on the bottom. The zero line on the top tells us where to look on the main scale: 0. Then we look for which of the top lines is aligned. It's 0 and 10. So this is precisely <u>zero</u>.

Figure 2: A measurement with a Vernier Compass

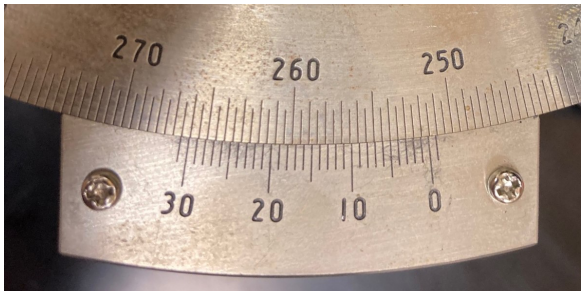

Figure 3: A 2nd measurement with a Vernier Compass

In figure 2, the main scale is the longer one, so it's on the top and the Vernier scale ranges from 0 to 30, so it is on the bottom. This is <u>zero</u> again.

In figure 3, the Vernier scale 0 points between 250 and 260 degrees. Looking closer, it is between 251 and 252 degrees. Looking closer, it's after the halfway mark, so it's more than 251 degrees and 30 minutes. (60 minutes = 1

degree.) Then we look at the Vernier scale. The line that matches up is 4 on the Vernier scale. So we add another 4 minutes. The answer is 251 degrees and 34 minutes.

0. Figures 4-11 on page 6 show measurements made with a Vernier scale, which measures to the nearest mm, and a Vernier compass, which measures to the nearest minute of arc, like the one we will use in this experiment. Choose 4 images (2 of the Vernier scale and 2 of the Vernier compass) and answer the following questions.
    a. What is the reading on the main scale?
    b. What is the reading on the Vernier scale?
    c. What is the total reading?
    d. For the Vernier compass readings, convert the measurement from degrees and minutes to radians.

Be sure to label which figures you chose in your solutions.

1. (Optional) You may wish to watch these videos demonstrating how to read a Vernier scale and Vernier compass.
    a. https://youtu.be/vkPlzmalvN4
    b. https://youtu.be/MnSg1yofYjc
    c. https://youtu.be/j0ARvWwMZX8?si=LuXTWaGT9xJ0aTfi

2. Compute the numerical value of the Rydberg constant, R, using the information in equations 2 and 3 of the lab handout. Please use the following values with high precision: $m_e c^2 = 510.999$ keV, $\hbar c = 197.327$ eV nm, $hc = 1239.8$ eV nm, and $\frac{e^2}{4\pi\epsilon_0} = 1.440$ eV nm. (As usual, using eV rather than SI will avoid a lot of stray powers of ten.)

3. Formula 3 allows us to predict what energy we might expect. We often discuss electron transitions in terms of different series, where each series is a different band of wavelengths (gamma, UV, visible, IR, microwave, etc). One series describes all the transitions to a final state of n_f=1, such as 2→1, 3→1, 4→1, etc. Another series describes all transitions to a final state of n_f=2, such as 3→2, 4→2, 5→2, etc.
    - Set up a spreadsheet that allows you to quickly calculate the energy and then wavelength for many transitions in the n_f=1, n_f=2, and n_f=3 series. Use high precision numbers.
    - In each case, calculate all transitions up to n_i=10.
    - Identify what part of the electromagnetic spectrum each series is part of. If it's visible, identify each color.

    Submit a printout of the Excel sheet that shows all these transitions and their wavelengths in nm.

INTRODUCTION

As far back as the early 19th century it was known that, when a pure material is heated so much that it becomes a glowing gas, the emitted light is composed of certain very definite wavelengths (colors). Since the advent of quantum mechanics, we have learned to interpret these patterns of wavelengths as arising from light emitted when electrons make transitions from one allowed atomic energy state to another. By measuring the emitted wavelengths of light, we learn about the allowed energies of electrons in atoms.

In this experiment, you will send light from a hydrogen gas lamp through a diffraction grating in order to determine which visible wavelengths are present. The diffraction grating is a series of closely spaced lines, etched in glass, which act like multiple slits. Light of wavelength λ falling on slits with spacing d will produce diffraction at angles θ given by:

$$d \cdot \sin\theta = m\lambda \qquad \text{where } m = \pm 1, \pm 2, \pm 3, \text{ etc.} \tag{1}$$

If many wavelengths are incident on the slits at the same time, then, according to eq. (1), the individual wavelengths will show constructive interference at different θs. A spectroscope can be used to measure the angles at which the different colors are seen, and, since we know the spacing between lines of the grating, will allow us to determine the wavelengths of the various colors in the spectrum.

Hydrogen is the simplest atom, consisting of one proton and one electron, and it has the simplest spectrum of all atoms and molecules. In the late 1800's, physicists found a remarkable empirical formula (*empirical*: based only upon the experimental evidence) that completely characterizes the various ultraviolet, visible, and infrared lines emitted by hydrogen. The formula is

$$\lambda^{-1} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{2}$$

where n_i is some initial state, n_f is some final state, and R is an experimentally obtained constant called "Rydberg's constant."

Early in the 20th century, the Bohr model explained the experimental hydrogen spectrum. In hot hydrogen gas, the atoms are usually in the $n = 1$ ground state and are moving rapidly. When two atoms collide with enough energy, an electron may be knocked to a higher orbit. A short while later, the electron will suddenly and spontaneously transition to a lower energy orbit, emitting a photon.

Because energy is conserved in this "quantum jump," and the electron loses energy as a result of a transition from a higher energy to a lower energy orbit, a photon must carry away the difference in energy E between these two energy levels. Einstein told us how to relate a unique photon energy E to a unique frequency (color, wavelength). Bohr's theory requires that any light emitted from hydrogen must be one of the following energies:

$$E = \frac{\mathrm{m}_e e^4}{2(4\pi\epsilon_0)^2\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{3}$$

where $n_i > n_f$ are positive integers. The transition involves a jump from the initial (i) to the final (f) orbit. In this way, Bohr completely explained the empirical formula (2), including obtaining an expression for R in terms of fundamental constants of nature. In class, we will learn that the Bohr model is too simplistic, but that Schrödinger's correct quantum mechanical treatment gives the *same results* for the energy levels of the Hydrogen atom.

THE EXPERIMENT

Measure the emitted wavelengths and analyze the hydrogen spectrum.

You will be able to see 4 visible lines in the first order ($m = 1$) and some of the lines in higher orders ($m = 2,3$) on both sides of the central image of the slit.

The spectrometer has been adjusted and focused, and the grating has been set as nearly as possible at right angles to the incident light. As you perform the experiment, you will adjust the **white and red knobs only**. These allow you to adjust the telescope position. The white knob is a set screw (a clamp) to loosen and tighten the telescope arm, and the red knob is for fine adjustment. Accidental use of the (many!) other knobs will result in a loss of alignment and an unhappy student.

Your job is to set the cross hairs (or pointer) of the telescope on each spectral line in turn (a flashlight through the slit can help you see the crosshairs/pointer better). Each position of the telescope will then be read on the circular scale with the aid of the vernier scale, and the angles can be recorded to the nearest minute of arc.

After the instructor has explained the spectroscope, you are ready to **observe the spectrum** from hydrogen gas, and to **make angle measurements for all lines that you can see**.

The zero-order (central pink) line should be fairly bright. If not, you may need to shift the lamp to the left or right a little.

Accounting for left-right misalignment:

1) **Center angle**. Check the angle marked for the zero-order (central pink) line. We will aim to set that to 0 degrees, but it can't be *perfect*. You can account mathematically for this by subtracting the center angle from your other measurements.
2) **Discrepancy**. You will likely STILL see a discrepancy between the measurement on the left side and the right side. We can mitigate this discrepancy by considering the left and right as a pair. Because of slight errors in alignment of the spectroscope, it is most accurate to determine the angle at which a spectral line occurs by measuring the angle of the line on the left and the corresponding angle on the right; then use
$$\lambda = \frac{1}{2}(\lambda_L + \lambda_R)$$
or equivalently $\sin\theta = \frac{(\sin\theta_l + \sin\theta_r)}{2}$,. This experiment is capable of both high accuracy and precision.
3) **Uncertainty**. Again, you should consider the left and right measurement as a **pair** that together give you one measurement. They are *not independent* measurements: if one is too far in one direction then the other will be too far in the other direction, and taking the average of the left and right wavelength gives you one good measurement. Since they are not independent measurements, you must **not** attempt to propagate the uncertainty through this average. You can estimate the uncertainty on each individual measurement, λ_L and λ_R, then use whichever uncertainty is larger for the uncertainty on the combined value.

General procedure:

1. Data. Record the angle readings in a well-organized table. You should measure all lines to the left and right of the center maximum ($m = 0$). Make sure that you record the angle of the center maximum as well. As with all measurements, estimate your uncertainty for each reading.
2. Calculate the wavelength and energy for each spectral line that you recorded above. An Excel or Sheets spreadsheet is helpful. We recommend that you use eV units with high precision: $m_e c^2 = 510.999$ keV,

$\hbar c = 197.327$ eV nm, $hc = 1239.8$ eV nm, and $\frac{e^2}{4\pi\epsilon_0} = 1.440$ eV nm. As usual, using eV rather than SI will avoid a lot of stray powers of ten.

3. Propagate uncertainty.
4. Graphical analysis.
    a. Verify that equation 2 works by creating a linear graph of your data. *Note: You will need to make educated guesses about the initial and final states, and then try them out.
    b. Include appropriate error bars on your datapoints.
    c. Determine the Rydberg constant *R* (in eV, with uncertainty) from your fit data and compare to the theoretical value you predicted in the prelab.

FUN STUFF

1. Move the eyepiece telescope away and just look at the bright lines of hydrogen without the lenses.
2. Use the hand-held spectroscopes and look out the window at the scattered sunshine (snow is good at scattering sunlight). The sun has hydrogen in the outer atmosphere, and that hydrogen ABSORBS the same colors that your hydrogen lamp EMITS. Can you see the dark absorption lines in the spectrum of the sun?
3. Your lamp might have areas that look like bright membranes. This is an example of the Frank-Hertz experiment. The lamp has a potential difference (a voltage) running from top to bottom, and in the vacuum chamber there is hydrogen and also there are electrons. The voltage is like a slide for the electrons, so they speed up, and when they reach a critical kinetic energy, they have enough energy to smash into hydrogen atoms and make them luminesce. Near the top and bottom of the lamp, that occurs at specific heights, which look like bright membranes.

LAB WRITEUP

Write results and discussion sections for the two-week combo of last week's numerical determination of energy levels and wavefunctions and this week's observations.

FIGURES FOR PRE-LAB QUESTION 1

Fig 4: A 2nd measurement with a Vernier Scale

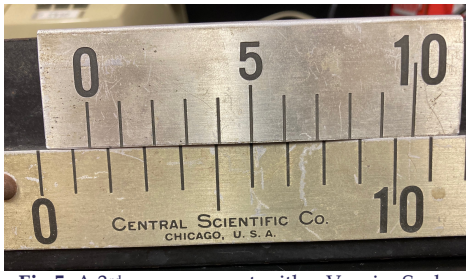

Fig 5: A 3rd measurement with a Vernier Scale

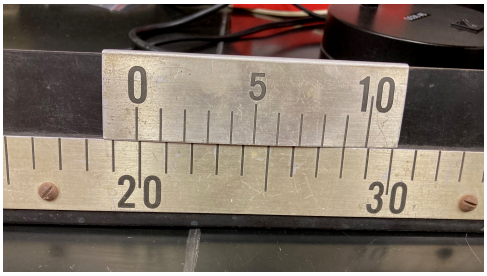

Fig 6: A 4th measurement with a Vernier Scale

Fig 7: A 5th measurement with a Vernier Scale

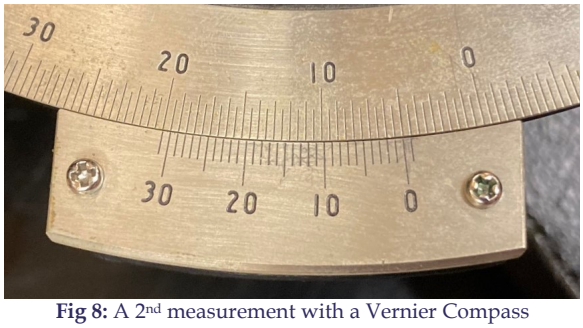

Fig 8: A 2nd measurement with a Vernier Compass

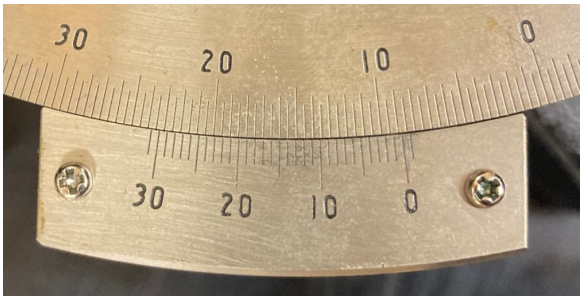

Fig 9: A 3rd measurement with a Vernier Compass

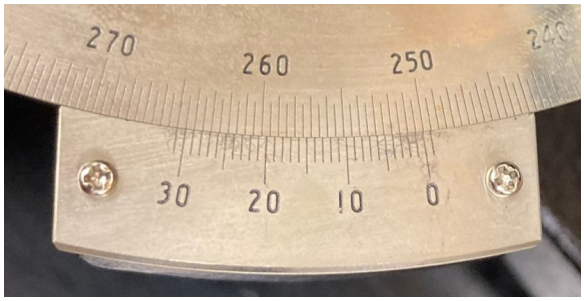

Fig 10: A 4th measurement with a Vernier Compass

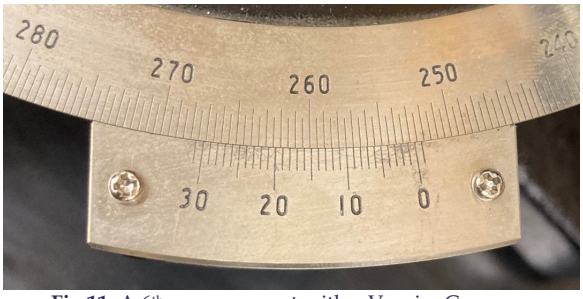

Fig 11: A 6th measurement with a Vernier Compass