Quantum (PHYS 290): Guide 11

Fall 2025 v1.5

Intro:

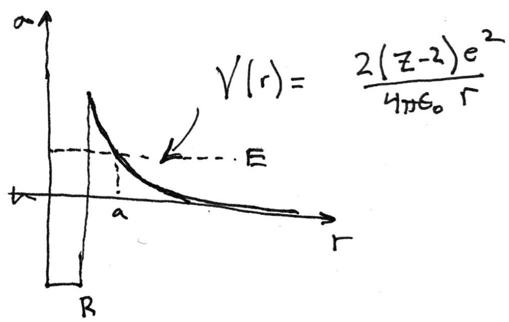
After looking at experiments that test our understanding of local realism - the two papers on the website are relatively readable - we have moved on to our last topic - multiparticle states and the structure of the periodic table.

Reading: ("T" stands for Townsend's text)

- Townsend's sections 7.2 -3
- We will have presentations on the context of the play Copenhagen starting Wednesday.

Problems: Due Friday, December 12 at the beginning of class

(1) (2 pts.) In radioactive alpha decay unstable nuclei release a $+2e$ charged ^{4}He nucleus (2 protons and 2 neutrons), or an "alpha particle". These alpha's have a mass of 3727 MeV/c^2 or about 4 atomic mass units. Before nuclear forces were better understood there was a bit of mystery about the lifetimes of the nuclei and the kinetic energies of the alpha particles. It turns out our work on tunneling (plus one more idea) explains the behavior. Let's assume that the potential of a nucleus for the positively charged alpha particle is



(a) Explain why this potential is reasonable. Be sure to include discussion of the well and the form of the potential outside the well.

(b) We found that the tunneling probability could be expressed as

$$T = e^{-G}$$

where G depends on both the width of the barrier and on the "wavenumber" κ. In this problem G is given by

$$G = \frac{2}{\hbar}\int_R^a \sqrt{2m(V(r) - E)}dr,$$

where $V(r)$ is given in the sketch. Notice that it depends on both the barrier width through the limits of integration and on the "wavenumber" through the usual dependence on κ. Find the value of a - the location where the alpha particle exits the barrier - as a function of the alpha's kinetic energy E. Show that

$$G = \frac{4}{\hbar}\sqrt{\frac{m(Z-2)e^2}{4\pi\epsilon_o}}\int_R^a \sqrt{\frac{1}{r} - \frac{1}{a}}\,dr$$

(c) The integration is

$$\int_R^a \sqrt{\frac{1}{r} - \frac{1}{a}}\, dr = \sqrt{a} \arctan\left(\sqrt{\frac{a}{R} - 1}\right) - \sqrt{\frac{R(a-R)}{a}} \simeq \frac{\pi}{2}\sqrt{a}.$$

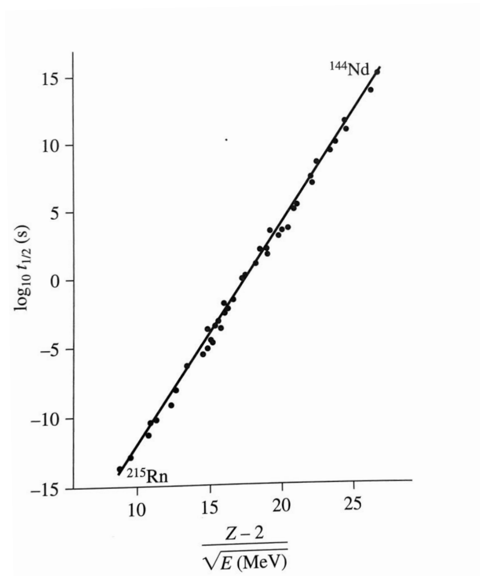
The approximation is valid for $R/a < 1$. Express G in terms of Z and E, when $R/a < 1$ and when E is in units of MeV.

(d) The second idea is this: The alpha particle has kinetic energy inside the well and bounces back and forth in the well. It repeatedly encounters the right wall of the potential, providing an opportunity to tunnel out of the well. The tunneling probability can then be expressed as the ratio of the time interval between encounters with the barrier and the lifetime of the nuclei τ,

$$T = e^{-G} = \frac{2R}{v\,\tau} \text{ or } \tau = \frac{2R}{v} e^{G}.$$

Here the speed is given by the alpha's kinetic energy inside the well. Sketch the log of the lifetime as a function of G. Interpret the slope and the y-intercept.

(e) Here's some data



Verify that the your solution (and the approximation) are correct. See what the y-intercept gives if you assume that the nuclei is R a few femtometers.

(2) 7.3 On where are these asymmetric and asymmetric particles states located?

(3) **OMIT** Particles are placed in a cubic box (or 3D infinite square well) with side length $L = 0.20$ nm. List the quantum numbers n_x, n_y, and n_z and find the lowest energies (in eV) of the system if

(a) eight noninteracting electrons are added to the box, and

(b) eight bosons of the same mass are added to the box.