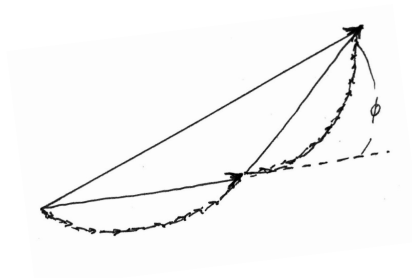
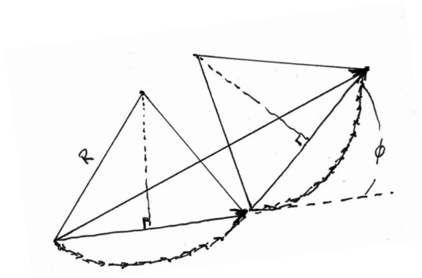
Solutions:

(1) Including single slit diffraction in Townsend's double slit intensity

(a) As we saw in class at an angle θ the wee little phasors from the Huygens' sources add up along an arc of a circle (since each phasor differs by a small angle $\Delta\varphi$[1]) to give the contribution from one of the finite-width slits. So we need to just add the arcs to the figure:



You could add the geometry with the radius R we used in class for each as well



but this is optional.

(b) We need to add up all the single slit Huygens' source phasors, the first one of which starts out with a phasor z_1 which we could write as $z_1 = r_s e^{i(kL-\omega t)}$, since the screen is L away from the slit. I will shorten this using $kL - \omega t = \alpha_s$. So for a single slit (s),

$$
\begin{aligned}
z_s &= z_1 + z_2 + z_3 + \cdots + z_N \\
&= r_s e^{i\alpha_s} + r_s e^{i(\alpha_s + \Delta\varphi_s)} + r_s e^{i(\alpha_s + 2\Delta\varphi_s)} + \ldots \\
&= r_s e^{i\alpha_s}\left(1 + e^{i\Delta\varphi} + e^{i2\Delta\varphi} + \ldots\right)
\end{aligned}
$$

[1]In the notation we used in class $\Delta\varphi = \pi\Delta y \sin\theta/\lambda)$ where Δy is the distance between adjacent sources.

(c) (Optional bonus worth +0.1 extra pt) The total final phasor z_P is the sum of z_s and another phasor shifted by the phase ϕ, $z_s e^{i\phi}$.

$$\begin{aligned}
z_P &= z_s + z_s e^{i\phi} \\
&= r_s e^{i\alpha_s}\left(1 + e^{i\Delta\varphi} + e^{i2\Delta\varphi} + \ldots\right) + r_s e^{i\phi} e^{i\alpha}\left(1 + e^{i\Delta\varphi} + e^{i2\Delta\varphi} + \ldots\right) \\
&= r_s e^{i\alpha_s}\left(1 + e^{i\Delta\varphi} + \ldots e^{i(N-1)\Delta\varphi}\right)\left(1 + e^{i\phi}\right) \\
&= 2 r_s e^{i\alpha_s} e^{i\phi/2}\left[\frac{\sin\frac{N\Delta\varphi}{2}}{\frac{N\Delta\varphi}{2}}\right]\left(\frac{e^{-i\phi/2} + e^{i\phi/2}}{2}\right)
\end{aligned}$$

In this last line I did some algebra to re-write in terms of sine and cosine in the expression; the last factor is cosine. Adding up the angles gives $N\Delta\varphi/2 = \pi a \sin\theta/\lambda$ so that

$$z_P = 2 r_s e^{i\alpha_s} e^{i\phi/2}\left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]\cos(\phi/2)$$

Finding the magnitude squared gives

$$z_P^* z_P = 4 r_s^2 \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2 \cos^2(\phi/2). \tag{1}$$

The double slit phase shift $\phi = 2\pi d \sin\theta/\lambda$ so this is the same as the intensity we derived in class (in a much easier way).

(2) The HeNe laser has $\lambda = 633$ nm.

(a) The screen is far way so we can use the small angle approximation

$$\sin\theta \simeq \theta \simeq \frac{\delta}{D} = 1 \times 10^{-3}.$$

The bright fringes (maxima) should be spaced so that

$$d\sin\theta = n\lambda \text{ or } d\frac{\delta}{D} \simeq \lambda \text{ for neighboring maxima.}$$

so

$$d = \frac{D}{\delta}\lambda = 10^{-3}\cdot 633 \text{ nm } = 633\times 10^{-6}\text{ m } = 0.633 \text{ mm}.$$

(b) The layer of cellophane adds phase so the whole pattern will shift. For instance the central maxima would not longer be at $\theta = 0$ but would be shifted to match the additional phase. Since the cellophane adds

$$\frac{\Delta\varphi}{2\pi} = 2.5 \implies \Delta\varphi = 5\pi$$

the position in the screen must change. The amount is given by this phase,

$$\Delta\varphi = \frac{2\pi d\sin\theta}{\lambda} = 5\pi$$

or, with the small approximation,

$$\frac{d\delta}{D\lambda} = \frac{5}{2} \implies \delta = \frac{3D\lambda}{d} = 2.5 \text{ cm}.$$

The central maxima would shift 2.5 cm. In the small angle approximation this holds for the other bright fringes as well.

(3) The stopping potential means that the electrons don't make it to the anode and the current stops. By energy conservation in this case, the total energy of the photon goes into the work function, to lift the electrons out of the material, and the stopping potential energy eV_o

$$hf = eV_o + W \implies W = hf - eV_o$$

(a) With the given numbers (sticking to eV helps here)

$$W = \frac{hc}{\lambda} - eV_o = 6.29 - 2.08 \simeq 4.21 \text{ eV}.$$

(b) The maximum kinetic energy is just the stopping potential energy so

$$K = eV_o \implies v = \sqrt{\frac{2eV_o}{m_e}} \simeq 8.55 \times 10^5 \text{ m/s}.$$

Although fast, this speed is well below the speed of light so SR is not needed.

(c) Intensity is number flux (here N_A per hour) times energy so

$$I = \frac{N_A}{3600s} \cdot \frac{hc}{\lambda} \simeq 169 \text{ W/m}^2.$$

(4) Since

$$hf = K + W \text{ or } K = hf - W$$

to find h using Millikan's data we need the slope on the K vs. f plot in figure 1.14. Choosing points from the plot $(0, 5.6 \times 10^{14})$ and $(2.35, 11.9 \times 10^{14})$ gives the slope

$$h = \frac{\Delta K}{\Delta f} \simeq 3.7 \times 10^{-15} \text{ eV s}$$

which differs the accepted value by about 10%.

(5) Photons with energy $E_\gamma = 0.66165$ MeV scattered off the target, loosing energy due to the recoil of the electrons. The data is given with a linear fit.

(a) With the Compton formula

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta) \text{ and } E = \frac{hc}{\lambda}$$

we have

$$\frac{hc}{E_{\gamma'}} = \frac{hc}{E_\gamma} + \frac{h}{m_e c}(1 - \cos\theta)$$

Multiplying through by E_γ and dividing by hc gives the result

$$\frac{E_\gamma}{E_{\gamma'}} = 1 + \frac{E_\gamma}{m_e c^2}(1 - \cos\theta).$$

(b) From this equation we expect that the line should be

$$1 + bx \text{ with } b = \frac{E_\gamma}{m_e c^2} = \frac{.0.66165MeV}{0.511MeV} \simeq 1.29.$$

The plot fit has $b = 1.4$. The experimental Compton wavelength is

$$\lambda_C = b\frac{ch}{E_\gamma} \simeq 2.6 \times 10^{-12} \text{ m}.$$

Meanwhile the theoretical value is

$$\lambda_C = \frac{h}{m_e c} \simeq 2.43 \times 10^{-12} \text{ m}.$$

(c) The agreement between the theoretical and experimental wavelengths is about 8%. From the plot we can see that the data doesn't look great. It doesn't look convincingly linear. Nor does the fit agree with the initial expected "1". The obvious next step is to take more data, particularly at large angle, above $\pi/2$. (But since this data run already took 3 weeks, we didn't.)

(6) A little radiation safety

(a) For instance a 1.5 GHz photon has energy $E = hf \simeq 6.2 \times 10^{-6}$ eV. A single photon of this energy would not break such a chemical bond.

(b) The situation is different for the photons used in the Compton experiment in problem 5, $\sim 7 \times 10^5$ eV. They are thus at an energy of 10^5 higher than typical bond energies and could do lots of damage. These photons were certainly dangerous so we took care to avoid these hot photons.

(7) 36,000 revolutions per inch ... Hmm so that means that the phase rotates 36,000 times over $d = 1$ inch. Since,

$$\frac{\Delta\varphi}{=} kd = \frac{2\pi d}{\lambda}$$

we have over $2\pi\, 36,0000$ radians

$$36,0000 \cdot 2\pi = \frac{2\pi d}{\lambda} \implies \lambda = \frac{2.54 \text{ cm}}{36000} \simeq 7.06 \times 10^{-7} \text{ m}$$

which is about 700 nm, which is the wavelength of red light.

(8) (2 pts.) Phasors in a thin film

(a) Because light reflects off the top and bottom we have to add two phasors. Let's suppose the accumulated phase of the phasor outside the film is α. The phasor for the reflected-off-the-top path is then

$$z_r = (0.2)(e^{i\pi})(e^{i\alpha}) = -0.2e^{i\alpha}$$

where the factors are amplitude for reflection (given), phase shift due to reflection, and accumulated phase. The phasor that goes through the thin film, z_t, is has an additional phase due to its travel through the film. This phase is based on the distance, $2d$, and the wavelength *in the material* λ'. So the additional phase is $\Delta\varphi = k'2d = 4\pi d/\lambda' = 4n\pi d/\lambda$. The phasor for the light that is transmitted on the top surface is[2]

$$z_t = (0.2)(e^{i\Delta\varphi})(e^{i\alpha}) = 0.2e^{i(\alpha+\Delta\varphi)}$$

The total amplitude is the sum of these

$$z = z_r + z_t = 0.2e^{i\alpha}\left(-1 + e^{i\Delta\varphi}\right) = 0.2e^{i\alpha}e^{i\Delta\varphi/2}\left(-e^{-i\Delta\varphi/2} + e^{i\Delta\varphi/2}\right)$$

This way of writing allows to rewrite the phase in terms of sine; $\sin\theta = (e^{i\theta} - e^{-i\theta})/2i$. So

$$z = (0.4)(ie^{i(\alpha+\Delta\varphi/2)})\sin(\Delta\varphi/2).$$

Taking the magnitude gives

$$P = z^*z = 0.16\sin^2\left(\frac{2n\pi d}{\lambda}\right)$$

which is equivalent to the probability stated in the problem.

[2] There is a problem. If the amplitude to be reflected is 0.2, then the probability of transmission must be $1-0.04 = 0.96$ so the amplitude of transmission is $\sqrt{.96} \simeq 0.98$. So really Townsend should quote the correct value or say the probability is approximate.

(b) Here we want a non-vanishing value of d for when $P = 0.16$, i.e when sine is one. Thus,

$$\frac{2n\pi d}{\lambda} = \frac{\pi}{2} \implies n = \frac{\lambda}{4d} \simeq 1.39$$

For the wavelength of 706 nm and $d = 5/10^6 * 2.54$ cm.

(c) For no reflection we need the two phasors to destructively interfere. This happens when the sine vanishes or when

$$\frac{2n\pi d}{\lambda} = \pi \implies d = \frac{\lambda}{2n} = 254 \text{ nm}.$$

(9) The rules are the same just *what* we are adding and multiplying changes. In classical probability we multiply and add probabilities. In quantum mechanics we multiply and add *amplitudes*.