

Intro:

Inspired by the approach to phasors and the clear interference effects in experiments with massive particles, we found Schrödinger's equation from conservation of energy. Since then we have developed both aspects of the wavefunction - at minimum it gives the probability density ψ^ψ - and solutions of the Schrödinger for particles in a box.*

In lab we ran the photoelectric effect experiment and this week we start a two week development of single photon interferometer.

Reading: ("T" stands for Townsend's text)

- T: Chapter 2 Sections 1, 3, and 4 are core for us. Please read 2.2 on Bragg scattering. We discussed light as a photon on Wednesday and Friday last week
- T: Chapter 2 Sections 6 - 9 are coming up this week. Just skim section 2.5 for the main point: Probability is conserved.
- T: Chapter 3 Sections 1 - 3 are core and we have discussed them already.
- T: Chapter 3 Section 4 - We'll likely get to this Wednesday next week.
- Looking ahead - Chapter 4 more 1D potentials and Schrödinger's equation.

Problems: Due Friday, October 3 at the beginning of class

- (1) 2.1 de Broglie wavelength for He
- (2) 2.2 He interference with uncertainties
- (3) 2.3 He single slit diffraction
- (4) The Planck length ℓ_P is the unique distance found by combining the reduced Planck's constant $\hbar$, Newton's gravitational constant G , and the speed of light c .
 - (a) Using dimensional analysis, find the expression for ℓ_P .
 - (b) Find ℓ_P in meters.
 - (c) How fast would a tennis ball have to go to have a wavelength of ℓ_P ? For inspiration see Example 2.1.BTW the Planck length is expected to be (roughly) the granularity of quantized geometry. If so then it would not be possible to build devices that have slits smaller than this size.
- (5) 1.34 The phasor work for a three slit pattern
- (6) 2.16 A linear combination of solutions is a solution...
- (7) 2.18 Finding where a particle is likely to be using ψ
- (8) 3.4 Time dependence for particle in a box.
- (9) Find the probability that a particle in a box of width L will be found in the interval $0 \leq x \leq L/4$ when it is in the $n = 2$ state.

- (10) (Optional 1 extra pt.) The diameters of the nearest stars are too small to be measured with the best grounded-based telescopes.¹ Brown and Twiss (Nature 178 (1956) 1046) proposed a method for such measurements and tested their method for the star Sirius. Using two old searchlight mirrors with photomultipliers at each focus, they measured the average value of product of the two currents. From the variation of this product with the separation of the two mirrors they determined the angle subtended by the star. Here's their schematic of the apparatus

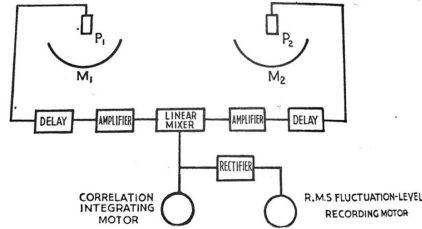
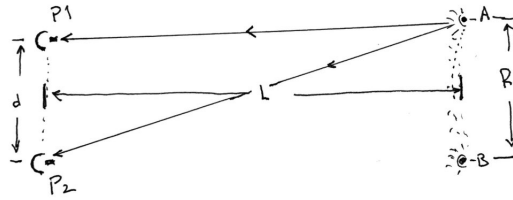


Fig. 1. Simplified diagram of the apparatus

(For more on this method see Sarah Stevenson's astrobite.)

There were at the time physicists who said that the method couldn't work since photons travel to either one mirror or another so there could be no correlation between currents coming from the detectors $P1$ and $P2$ (such as we use in lab) and shown in the diagram. Let's explore why this argument is incorrect. Here's the geometry including the detectors and the star. Clearly this is not drawn to scale!



The Brown-Twiss apparatus is on the left. I have reoriented the experiment and labeled their detectors $P1$ and $P2$. There are two light sources at A and B ; think about them as the edges of the star of radius R . They are at large distance L from the detectors. These detectors are connected to a circuit that measures the simultaneous arrival of photons.

- (a) Show that

$$\frac{1}{2\pi} \int_0^{2\pi} e^{i\theta} d\theta = 0.$$

- (b) Let z_1 be the amplitude for a photon to arrive at the detector $P1$, x_{1A} be the distance between the source A and the detector $P1$, and x_{1B} be the distance between the source B and the detector $P1$. The light from these two sources is not coherent so there is an additional random phase θ between the two sources. Explain why the phasors z_1 and z_2 can be written as

$$z_1 = r \left(e^{ikx_{1A}} + e^{i(kx_{1B} + \theta)} \right)$$

and

$$z_2 = r \left(e^{ikx_{2A}} + e^{i(kx_{2B} + \theta)} \right).$$

¹The diameter of a star was first measured by Michelson using a large optical interferometer.

- (c) The magnitude squared of z_1 is

$$|z_1|^2 = r^2 \left(2 + e^{i(k\Delta x_1 - \theta)} + e^{-i(k\Delta x_1 - \theta)} \right)$$

Since the measurements were taken over some time this averaged over the relative phase. Using part (a) show that the averaged magnitude squared is

$$\langle |z_1|^2 \rangle = 2r^2.$$

- (d) Brown and Twiss looked for coincidence counts which are proportional to $\langle |z_1|^2 |z_2|^2 \rangle$. Show that this average is

$$\langle |z_1|^2 |z_2|^2 \rangle = 2r^4 (1 + \cos [k(\Delta x_1 - \Delta x_2)]).$$

- (e) Assuming $L \gg d$ show how can this result be used to measure R if d , λ and L are known.