

Intro:

We now have a collection of cases for energies and wavefunctions for 1D potentials, using “Schrödinger + boundary conditions”. This week we will look again at the formalism of quantum mechanics, starting with the definition of operators, eigenvalues and eigenstates. This material in Chapter 5 includes a careful study of the Heisenberg uncertainty relations.

Reading: (“T” stands for Townsend’s text)

- T: Chapter 5 Sections 1 - 4.
- Looking ahead - Chapter 5 Sections 5 - 6

Problems: Due Friday, October 31 at the beginning of class

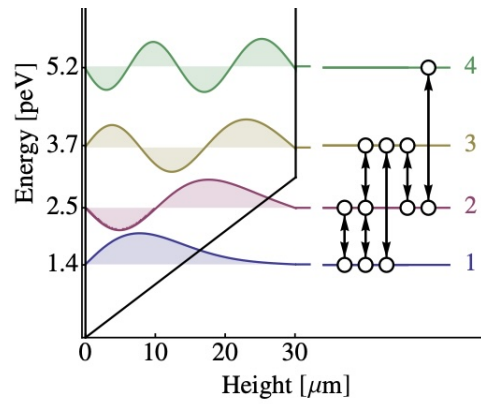
- (1) Part I: Solving the quantum bouncing ball!
 - (a) Write down the potential energy for a ball bouncing off a floor in Earth’s gravitational field. Let’s choose $x = 0$ at the floor and “up” being the positive x direction.
 - (b) Show the TISE can be expressed as

$$\frac{d^2u}{dz^2} - zu = 0$$

where z is defined as

$$x = az + b \text{ with } a^3 = \frac{\hbar^2}{2m^2g} \text{ and } b = \frac{E}{mg}.$$

- (2) (2 pts.) Part II: Solving the quantum bouncing ball! Using the techniques we learned in lab, solve the four lowest energies and associated wavefunctions for the bouncing ball potential. Since the ‘0’ boundary condition is at the origin, it is helpful to solve from “right to left” (high z to low z) rather from “left to right” as we did in lab. In your solutions include the dimensionless energy b/a for the different energy levels and the plots of the (unnormalized) wavefunctions.
- (3) In a series of experiments Abele and collaborators at the Technical University in Vienna have bounced neutrons off a floor or “neutron mirror”. This is used to test whether there are any deviations from Newtonian gravity (such a chameleon dark energy fields). The neutrons have a little horizontal velocity so that bounce through the apparatus - or would do so if it was a classical experiment. In a 2014 experiment, neutrons bounced off the mirror, while above they had an upper plate that absorbed neutrons, skimming off those neutrons with the highest energies, leaving only neutrons in a selected quantum state. In the following plot from their 2015 paper (Jenke et al, Physical Review Letters, 112 (2014) 151105) they show energies, wavefunctions, and observed transitions.



Using your work see how far you can get in explaining the various features of this plot. Start with qualitative features. The mass of neutrons is linked [here](#). (Thank you NIST!)

- (4) 5.1 Hermiticity of the parity operator
- (5) 5.2 and (b) show that momentum is!