Quantum (PHYS 290): Guide 8

Fall 2025 v1.0

Intro:

We're in the middle of exploring the formalism of quantum mechanics, with Hermitian operators, eigenvalues and eigenstates. Using this we'proved the Heisenberg uncertainty relations based on the non-commutivity of momentum and position. This week we move on to solving quantum problems in 3D and the hydrogen atom.

Reading: ("T" stands for Townsend's text)

- On Monday we will discuss the radial function of the Hydrogen atom (Chapter 6 Section 3) - in preparation for lab.
- Later in the week we will start on Chapter 6, Sections 1 - 2
- So far we have discussed in Chapter 5 Sections 1 - 4. We will return to Section 6 later. We will skip Section 5.
- Looking ahead - Chapter 6 Sections 4 - 6, Chapter 5 Section 6.

Problems: Due Friday, November 7 at the beginning of class

(1) Find which of the following functions are eigenfunctions of the momentum operator

$$\hat{p} = -i\hbar\frac{d}{dx}$$

If the function is an eigenfunction find the eigenvalue.

$$u_1 = \sin(kx),\ u_2 = e^{-ik(x-a)},$$

$$u_3 = \sin(kx) - \cos(kx),\ u_4 = \cos(kx) + i\sin(kx)$$

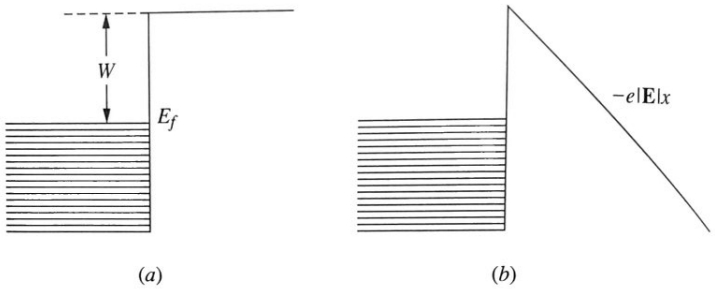
(2) 5.9 Hermiticity and $\left\langle \hat{A}^2 \right\rangle$

(3) 5.11 Finishing a bit of the uncertainty argument

(4) Using the notation we had in class,
   (a) show that $\left\langle \hat{D}_A^2 \right\rangle = \langle \Delta A \rangle^2$ and
   (b) $\left[\hat{D}_A, \hat{D}_B\right] = \left[\hat{A}, \hat{B}\right]$.

(5) Electrons bound in a metal can be modeled by the potential shown in (a) below.[1] The highest energy of the electrons is E_f. The work function is W. When photons with energies of W or higher are absorbed by electrons, the electrons are ejected from the metal - the photoelectric effect. In this problem we explore another way to pull electrons out of a metal - applying a large enough electric potential as shown in (b). In this second case, the electrons tunnel through the triangular potential barrier.

[1] Image from Townsend's other quantum book



(a) Show that the potential energy shown in (b) can be written as

$$V(x) = E_f + W - e|\mathbf{E}|x$$

where $\mathbf{E}$ is the electric field and e is the fundamental charge.

(b) Using the approximate tunneling transmission

$$T \simeq \exp\left(-\frac{2}{\hbar}\int \sqrt{2m(V(x)-E)}dx\right)$$

show that for electrons at energy E_f

$$T \simeq \exp\left(-\frac{4\sqrt{2mW^3}}{3e|\mathbf{E}|\hbar}\right).$$

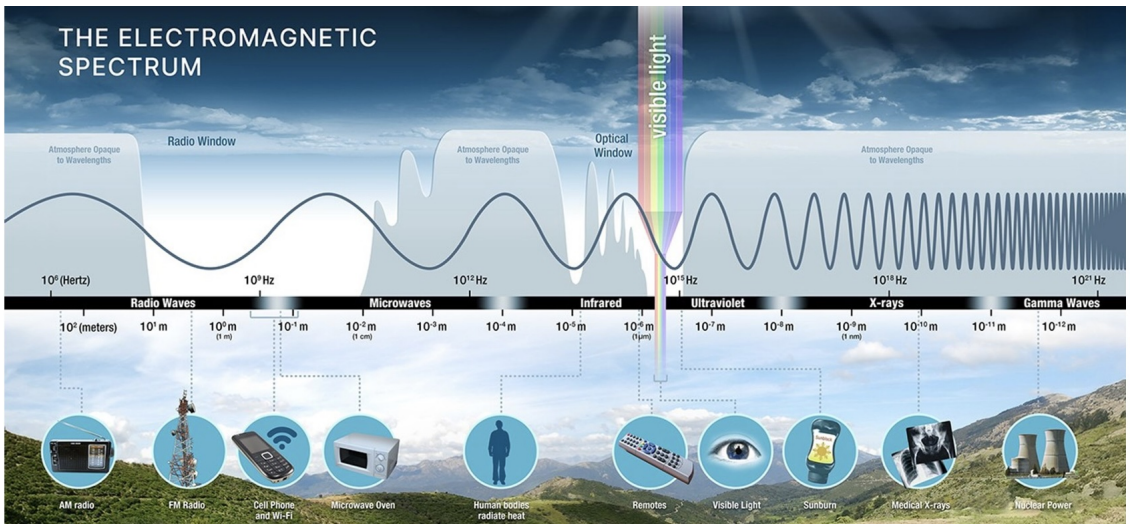
(6) Hydrogen energy levels

(a) Find the energies of the $n = 1, 2, 3$ states in electron volts.

(b) Calculate the frequencies in Hz and the wavelengths in nm of all the photons that can be emitted from the atom in transitions between these levels.

(c) Sketch a energy level diagram with the transitions labeled.

(d) In which part of the electromagnetic spectrum are these photons located?



Thanks to NASA, Science Mission Directorate, “Introduction to the Electromagnetic Spectrum”, website (2010)