Quantum (PHYS 290): Midterm II Practice

Fall 2026 v1.0

This quiz is under the auspices of the Hamilton Honor Code.

- You may use a calculator.
- Please ask questions, particularly when the statement of the question is confusing.
- Other than me, you may not consult with any person, text or resource via any means.
- You have 3 hours to complete your solutions.
- There are 4 questions and a total of N possible points.
- May the light be with you!

1. Some of our favorite expressions

Constants:

$$h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s}$$
$$\hbar = h/2\pi = 1.05 \times 10^{-34} \text{ J s } = 6.58 \times 10^{-16} \text{ eV s},$$
$$m_e = 9.11 \times 10^{-31} \text{ kg } = 0.511 \text{ MeV/c}^2, \; m_p = 1.67 \times 10^{-27} \text{ kg } = 938 \text{ MeV/c}^2$$
$$c = 2.998 \times 10^{8} \text{ m/s}$$
$$e = 1.60 \times 10^{-19} \text{ C},$$

Trig Expressions:

$$e^{i\theta} = \cos\theta + i\sin\theta, \text{ so } \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \text{ and } \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos^2\theta = \frac{1}{2}\left(1 + \cos 2\theta\right) \text{ and } \sin^2\theta = \frac{1}{2}\left(1 - \cos 2\theta\right)$$

A phasor is

$$z = re^{i(kx \pm \omega t)} \text{ typically we time average and use } z = re^{ikx}$$

The probability is

$$P = |z|^2$$

The intensity I,

$$I = |z|^2$$

for light emitted isotropically with power P_o,

$$I = \frac{P_o}{4\pi r^2}$$

Interference and diffraction for double and single slits

$$d\sin\theta = n\lambda, \; a\sin\theta = m\lambda$$

$$I(\theta) = I_o \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

Waves:

$$v = \lambda f, \; v = \frac{\omega}{k}, \; k = \frac{2\pi}{\lambda} \text{ and } \omega = \frac{2\pi}{T}$$

In a material with index of refraction n, the speed of light is $c_n = c/n$ and $k_n = nk$.

$$\vec{E}(x,t) = \vec{E}_o \sin(kx - \omega t)$$

Photons

$$E_\gamma = hf = \hbar\omega = \frac{hc}{\lambda} \text{ and } p_\gamma = \frac{h}{\lambda}$$

Uncertainty: For addition and subtraction, if $q = x \pm y$ then $\delta q = \sqrt{\delta x^2 + \delta y^2}$. For multiplication and division then, if $q = x/y$ then add the relative uncertainties in quadrature

$$\frac{\delta q}{q} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}$$

For a general calculated quantity $q = q(x, ..., z)$ then

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + ... + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$

2. Problems:

(1) (15 pts.) An electron is inside a "particle in a box" potential with width 1.00 nm.
   (a) Sketch the wavefunctions for the $n = 1$ and $n = 2$ states.
   (b) Sketch the probability densities for the $n = 1$ and $n = 2$ states.
   (c) Find the energy of the $n = 2$ state.
   (d) For the $n = 1$ state find the probability of finding the electron between 0.15 nm and 0.35 nm where $x = 0$ is the left side of the box.
   Hint: I found the integral

$$\int_{0.15}^{0.35} \sin^2(\pi x)dx = 0.1$$

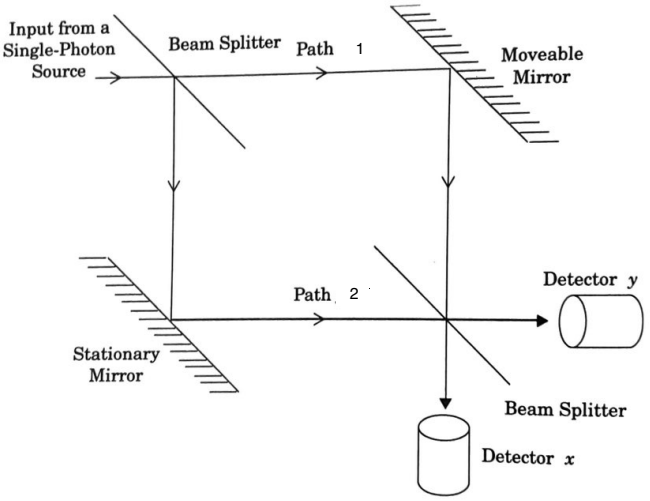
   useful!

(2) (25 pts.) Short answer questions
   (a) Given a frequency like 56 MHz what is the wavelength of this "light"?
   (b) Can single photons interfere? Which experiment tests this?
   (c) Walter Huckel wrote (translation by Felix Bloch)

   Erwin with his ψ can do,
   Calculations quite a few.
   But one thing has not been seen
   Just what does ψ really mean?

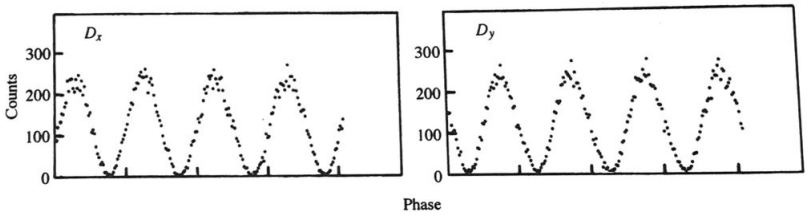
   What does the wavefunction mean? What do you do with it?
   (d) If you have an energy eigenstate $\psi_n(x)$ then what is $\psi_n(x, t)$?
   (e) How can we create (mathematically) a localized wave packet for a particle?

(3) (20 pts.) Consider the following Mach-Zehnder interferometer



Let the distance along path 1 from the first beam splitter (next to the source) to the last beam splitter be d_1. Notice that this path has the moveable mirror. Similarly, let d_2 be the distance along path 2 from first to last beam splitters. Assume that the distances are the same from the final beam splitter and the detectors.

(a) Find the four phasors for the four possible paths from the first beam splitter to the detectors. For instance z_{1x} is the phasor for path 1 ending at the x detector.
(b) Find the probabilities of detecting a photon at detector y and detector x
(c) Here's the data from the experiment



"Phase" is proportional to the distance the mirror moves. Compare your prediction from part (b) to this data, including discussion of the shape and the relative phase.

(4) Consider the following state in a quantum simple harmonic well

$$\psi = \frac{\sqrt{3}}{2}\psi_o + \frac{1-i}{2\sqrt{2}}\psi_1$$

where ψ_n are the normalized states for the energies $E_n = \hbar\omega(n + 1/2)$, $n = 0, 1, 2, \ldots$.
(a) What is the probability of obtaining $\hbar\omega/2$ when a measurement in this state is made?
(b) What is the expectation value for energy in this state?
(c) Determine the time dependence of ψ.
(d) Find ΔE in this state at time t.