Error Analysis in a Nutshell

Systematic vs. Statistical (Random) Errors: (Taylor 4.1, 4.6)

Statistical (Random) errors can be reduced by repeating a measurement so the random fluctuations average to zero. Statistical techniques (Std. Deviation, Std. Error, Error Propagation, etc.) apply to statistical errors.

Systematic errors are some difference between our model of the system and the physical system. They are reproducible so they do not average to zero. They can't be improved by repeated measurements and each case must be handled differently.

Uncertainty in a single parameter (Taylor 1.5, 2.1-2.3, 2.8)

Single Measurement:

Estimate the believable extent of your measurement as best you can.
(This is likely not be the same as the displayed tick marks! See pages 9 and 10 in Taylor for examples.)
Sig Figs! Normally one digit uncertainty. Quote value to same precision as δ.

Multiple measurements:

Standard Deviation: (Taylor 4.2-4.3)

The standard deviation gives the expected uncertainty for a single measurement. It also describes the scatter in values from many repeated measurements. Use STDEV() in Excel or a calculator.

Standard Error: (Taylor 4.4)

The standard error is the uncertainty in the average of repeated measurements. You would expect the average of N measurements to be more accurate than a single measurement, but for a random process, 2 measurements is not twice as good as 1 measurement. Repeating a measurement N times reduces your uncertainty by a factor of $\frac{1}{\sqrt{N}}$.

$$\text{Std. Error} = \frac{1}{\sqrt{N}}\ \text{Std. Dev.}$$

For a slope:

use the LINEST function in Excel
Select a two-by-two grid
=LINEST(y values, x-values, true, true)
(first "true": also fit intercept, second "true": calculate errors)
Control-shift-return

Slope	y-intercept
Slope uncertainty	y-intercept uncertainty

Error Propagation: (Taylor 3.5-3.8)

Errors can be propagated through any equation using only three rules.

- For $q = x+y$ or $x-y$, $\delta q = \sqrt{\delta x^2 + \delta y^2}$
- For $q = xy$ or x/y, $\frac{\delta q}{q} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}$
- For $q = f(x)$, $\delta q = \left|\frac{df(x)}{dx}\right| \delta x$

For more complex equations, these rules can be applied in steps.

For $q = (x+y)*z$, find the uncertainty in the quantity $(x+y)$ first, then use the product rule to combine $(x+y) \pm\sigma_{(x+y)}$ times $z \pm\sigma_z$.

Uncertainty Review Guide

Taylor, "An Introduction to Error Analysis."

Significant Figures:		
Basic Sig Figs:	Sect 2.1-2.3	
Uncertainty more accurate than sig figs:	Sect 2.8	
Propagation of errors:		
Add/Subtract:	Sect. 3.5	Eqn. 3.16
Multiply/Divide:	Sect. 3.6	Eqn. 3.18
Function of 1 Variable:	Sect. 3.7	Eqn. 3.23
"Chain Rule", Dominant Errors:	Sect. 3.8	
Standard Deviations:		
Counting Statistics:	Sect. 3.2	Eqn. 3.2
Standard Deviation:	Sect. 4.2, 4.3	use calculator
Standard Error (Stand. Dev. of the Mean)	Sect. 4.4	Eqn. 4.14
Systematic Errors:	Sect. 4.1, 4.6	
More Advanced Topics:		
Normal distribution, confidence limits	Ch. 5	
Rejecting Data:	Ch. 6	
Weighted Average:	Ch. 7	
And more!	Ch. 8 - 13	