1/18/23

source of electric force is <u>charge</u>: q, Q

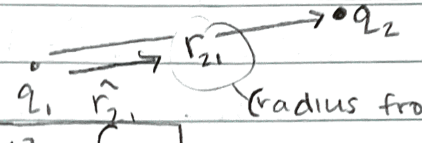
- charge is quantized in units of $e = 1.6 \times 10^{-12}$ C ← coulomb (SI unit of charge)

$q_e = -e$ "large amount of energy"

$q_p = e$

★this semester: q is continuous + conserved (path independant)

coulomb force



$$\vec{F}_2 = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

permitivity of free space

ϵ_0: permitivity of

$$\boxed{\text{freespace: } 8.85 \times 10^{-12} \frac{C}{N m^2}}$$

$k = \frac{1}{4\pi\epsilon_0}$: (can treat as constant: $8.988 \times 10^9 \approx 9 \times 10^9 \frac{Nm^2}{C^2}$

↳ compared to gravity: $\vec{F_g} = \frac{-GmM}{r^2} \hat{r}$

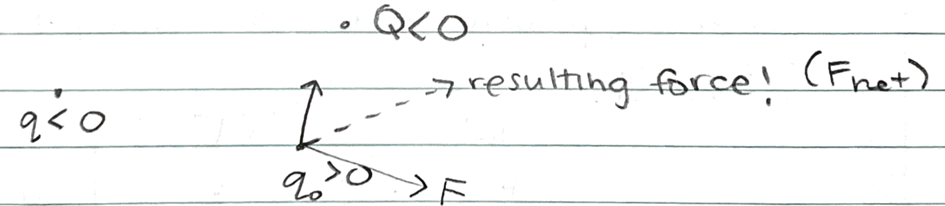
m's ALWAYS positive

vs. q's have both sides = +, −

coulomb's force = $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$

⇒ SATISFIES SUPERPOSITION

ex:



ENERGY what is the energy (PE) in a configuration of charges?

q_1 is for free - no work required

q_2: how much work is required to assemble q_1, q_2 configuration

Far away (∞)

★putting origin at q_1 q_1 — r — · $\overleftarrow{d\vec{r}}$ $\longrightarrow \vec{F}$ (coulomb) q_2

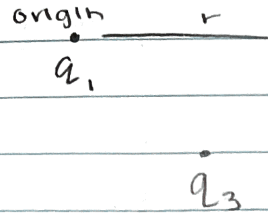
$$W = -\int \vec{F} \cdot d\vec{s} = -\int_{\infty}^{r} \vec{F} \cdot d\vec{r}$$

negative of coulomb F

$$= -\int_{\infty}^{r} |F| dr = -\int_{\infty}^{r} \frac{q_1 q_2}{4\pi\epsilon_0 r^2} dr = \frac{q_1 q_2}{4\pi\epsilon_0} \int_{\infty}^{r} \left(\frac{-1}{r^2}\right) dr$$

config for a single charge $= \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r}\right)\Big|_{\infty}^{r} = \frac{q_1 q_2}{4\pi\epsilon_0 r}$

example continued w: q_3: $W = -\int \vec{F}_3 \cdot ds = \int |\vec{F}_{31} + \vec{F}_{32}| \cdot d\vec{s}$



$$= -\int \vec{F}_{31} \cdot d\vec{s} - \int \vec{F}_{32} \cdot d\vec{s}$$

(any path chosen will give the same radial independance(e))

just add together to get: ↙

config for two charges $= \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1 q_3}{r_{31}} + \frac{q_2 q_3}{r_{32}}\right) \Rightarrow$ electric potential just adds like a scalar

→ suppose we have "n" charges:

$$U = \frac{1}{4\pi\epsilon_0}\underbrace{\left(\sum_{j=1}^{N}\sum_{k<j}^{N} \frac{q_j q_k}{r_{jk}}\right)}_{\text{counting for all the pairs + radaii}}$$

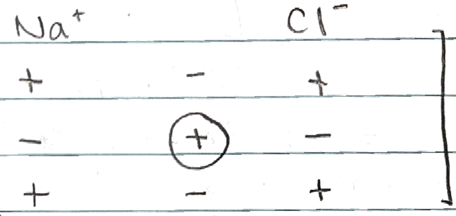
config for any # of charges $= \boxed{\left(\frac{1}{2}\right)\left(\frac{1}{4\pi\epsilon_0}\right)\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N} \frac{q_j q_k}{r_{jk}}}$ → double counting and dividing by 2

$\boxed{1/20/23}$

review: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}$ (COULOMB'S LAW)

→ energy: work done to assemble config of charges

$\boxed{\text{example}}$



simple crystal, everything is "a" distance apart

for circled Na^+:

$$u = \left(\tfrac{1}{2}\right)\left(\frac{1}{4\pi\epsilon_0}\right)(e)\left(\frac{-6e}{a} + \frac{12e}{\sqrt{2}a} - \ldots + \ldots\right)$$

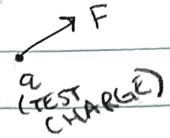
↳ add N to get any config: ↑ if mapping out cont

$$u = \left(\tfrac{1}{2}\right)\left(\frac{1}{4\pi\epsilon_0}\right)(Ne)\left(\frac{-6e}{a} + \frac{12e}{\sqrt{2}\,a} + \ldots - \ldots\right)$$

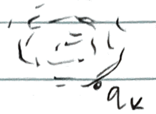
ELECTRIC FIELDS (GAUSS' LAW)

1/20/23

$$\vec{E} = \frac{\vec{F}}{q}$$



→ ways of looking at an E-field



weaker charge greater charge

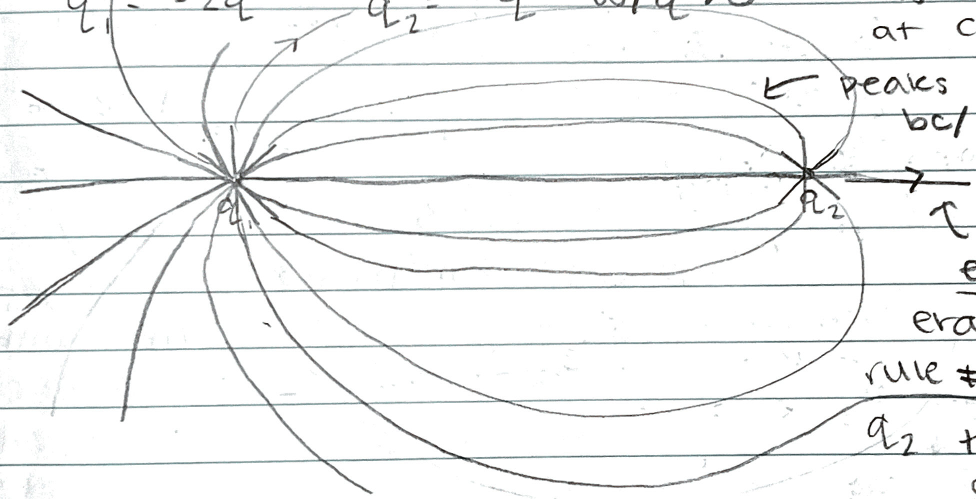
⁎ < ✳

- field lines:
  - # of lines is proportional to the charge
  - density of lines is proportional to field strength,
  - direction is tangent to the E-field

example of drawing e-fields:

$q_1 = -2q$ $q_2 = +q$ w/ $q > 0$

★ no idea what is going on at charge



← peaks over here bc/ $q_2 < q_1$ (WEAK HERE)

↑ this can't exist, so either erase and break rule #1, or rotate q_2 to achieve symmetry

So $|\vec{E}| = 0$ when, $\left(\cancel{\frac{1}{4\pi\epsilon_0}}\right)\left(\frac{-2\cancel{q}}{x^2} + \frac{\cancel{q}}{(x+d)^2}\right) = 0$

→ let d = 1m

$$\frac{-2}{x^2} + \frac{1}{(x-1)^2} = 0$$

$$x^2 = 2(x-1)^2$$

$$x^2 = 2x^2 - 4x + 2$$

$$0 = x^2 - 4x + 2$$

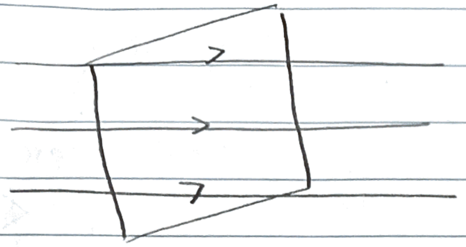
→ use quadratic: $x = \frac{-4 \pm \sqrt{16-8}}{2} = 2 \pm \frac{\sqrt{8}}{2}$

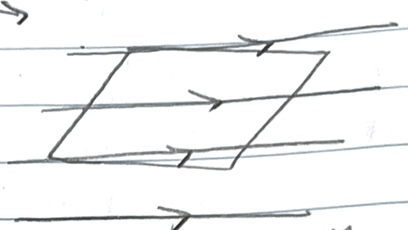
$\boxed{x = 2 \pm \sqrt{2}}$ where the field is zero

↳ must be: $x = 2 + \sqrt{2}$

flow of water:

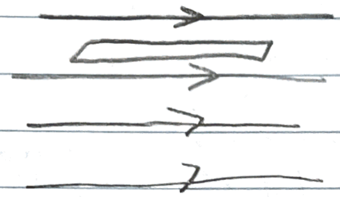
tilting board at an angle →





LOTS PASSES THROUGH

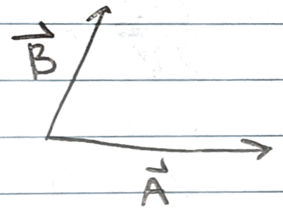
LESS PASSES Through



surface is perpendicular to the flow of water

⇒ $\boxed{\text{FLUX}}$

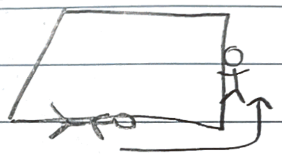
NONE PASSES THROUGH
(dot-product)



(perpendicular to surface AB

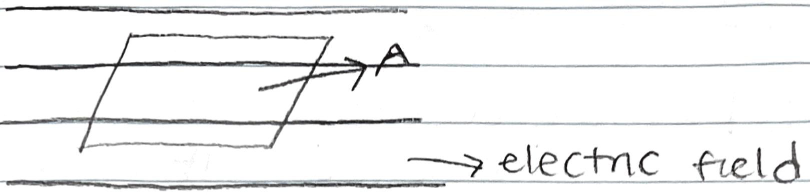
$$\vec{a} = \vec{A} \times \vec{B}$$

$$|\vec{a}| = |\vec{A}||\vec{B}|\sin\theta$$

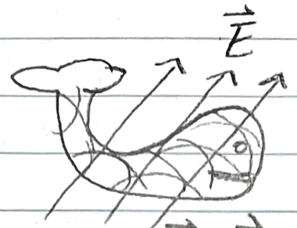


(if walking around a perimeter, leave the area, ALWAYS to the left, area direct is from foot to head

Flux Φ dot-product involved → $\Phi = \vec{E}\cdot\vec{A}$ 1/23/23



more complex:



Φ through the surface:

$\Phi_1 = \vec{E}_1\cdot\vec{A}_1$ (first surface)

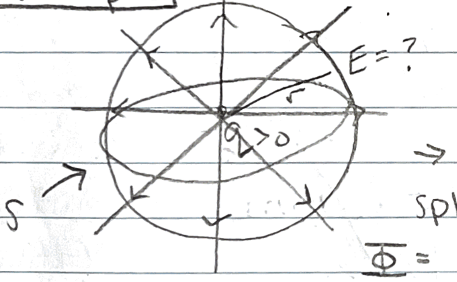
so through whole surface

$$\Phi = \Phi_1 + \Phi_2 + \dots = \vec{E}_1\cdot\vec{A}_1 + \vec{E}_2\cdot\vec{A}_2 + \dots = \sum_{j=1}^{N} \vec{E}_j\cdot\vec{A}_j$$

(w/"N" small surfaces)

→ goes to $\int \vec{E}\cdot d\vec{a} = \Phi$ as $N \to \infty$

example what is $\vec{E}$ at distance "r"?



$$E = \frac{\vec{F}_e}{q_{test}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

→ what is the flux through a sphereical surface of radius r?

$\Phi = \int \vec{E}\cdot d\vec{a}$ ★ $\vec{E}$ is radial

$= \int E\,da$ ★ $d\vec{a}$ is radial

method

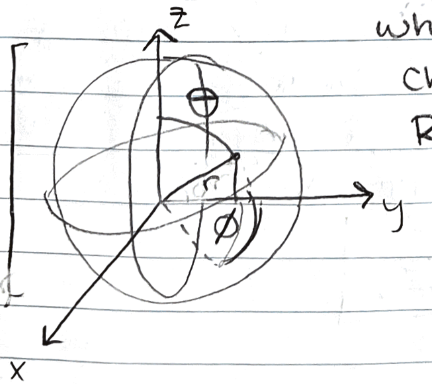
-direction of $\vec{E}$?

-dot product?

-find da → find E

$= \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\, E r^2 \sin\theta$

spherical co-ord review:



a larger/smaller sphere will have same flux, also tilting the shape will NOT change it.

↳translates x,y,z to r, ϕ, θ

when changed by dθ → distance changes by rdθ

$R = r\sin\phi\, d\phi$

$da = r^2 \sin\phi\, d\theta\, d\phi$

$r^2 E (2\pi) \int_0^{\pi} \sin\theta\, d\theta$

$= \boxed{2\boxed{(2\pi r^2 E)}} = \boxed{4\pi r^2 E}$

with coulomb's law:

$$4\pi r^2 \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{q_{enclosed}}{\epsilon_0}$$

→ flux through surface is the same regardless of shape → except for if perpendicular

$\boxed{\text{example 2:}}$ multiple charges enclosed in sphere

↳ w/ $q_1, q_2 \ldots q_n$, what is E?

$$\Phi = \oint_S \vec{E}\cdot da = \underbrace{\int \vec{E}_1 \cdot da}_{\text{flux to Charge 1}} + \underbrace{\int \vec{E}_2 \cdot da}_{\text{flux to charge 2}} + \underbrace{\int \vec{E}_3 \cdot da}_{\text{flux to charge 3}} + \ldots$$

$$= \frac{q_1}{E_0} + \frac{q_2}{E_2} + \ldots = \boxed{\oint E_1 d\vec{u} = \frac{q_{enclosed}}{\epsilon_0}}$$

better way to find E w/ situations of symmetry

{GUASS' LAW}

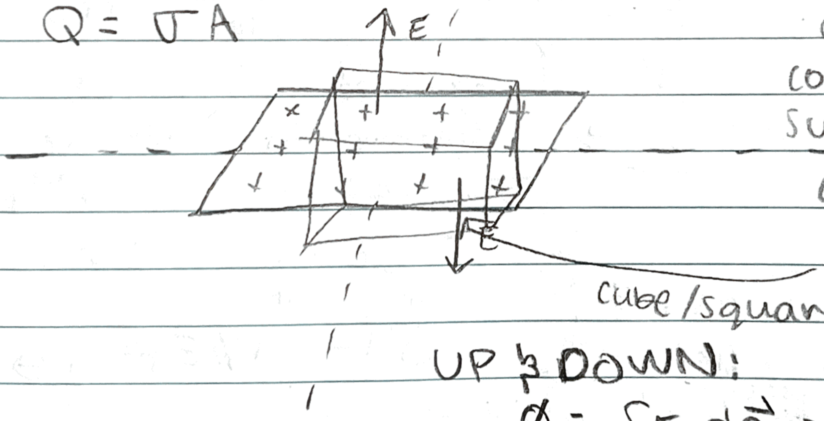
$\boxed{\text{example}}$

(useful for planar, cindrical, or sphereical symm)

→ suppose you have uniformly charged ∞ plane

$0 < \sigma$: area charge, density

$Q = \sigma A$



choose convienent surface: GAUSSIAN PILL BOX

cube/square OR cylinder

UP & DOWN:

$$\emptyset = \int E \cdot d\vec{a} = E\int da = EA$$

→ no flux through side = perpendicular

total flux = $\emptyset_{top} + \emptyset_{bottom} + \cancel{\emptyset_{sides}} = \boxed{2EA}$

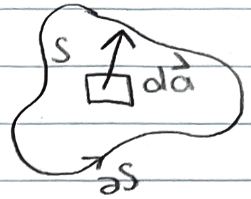
charge enclosed: σA

$$\boxed{\text{e-field: } E = \frac{\sigma}{2\epsilon_0}}$$

QSR hours
T: 5-9
W: 12-3
R: 12-4
Su: 4-9

1/25/23

electric flux: $\Phi_E = \int \vec{E}\cdot d\vec{a}$



GAUSS' LAW

$$\boxed{\oint \vec{E}\cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}}$$

* useful for computing $\vec{E}$ in: planar, sphereical, & cylindrica

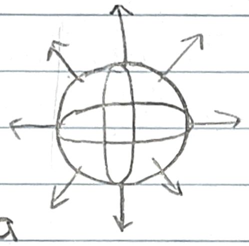
GAUSS' LAW practice

method
- determine direction of $\vec{E}$
- chose "Gaussian pill box"
- compute LHS & RHS
- find $|\vec{E}|$
- sketch $|\vec{E}|$, finalize $\vec{E}$

* σ: area charge density
ρ: volume charge density
λ: linear charge density

→ spherical
- hollow sphere w/ surf charge density σ and radius R, find $\vec{E}$ everywhere

$Q = \sigma A$



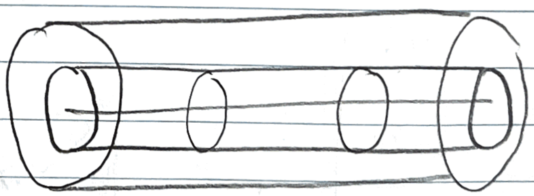
$q_{encl} = \int \sigma\, da$

$r > R \longrightarrow \vec{E} = 0$

$r < R$: $q_{encl} = \sigma(4\pi r^2)$

$\oint \vec{E}(4\pi r^2) = \frac{\sigma 4\pi R^2}{\epsilon_0}$

$$\boxed{\vec{E} = \frac{\sigma R^2}{\epsilon_0 r^2}}$$



→ cylindrical
- a long solid cylinder w/ volume charge ρ, find $\vec{E}$ at any r.

$\oint \vec{E}\cdot da = \underbrace{\cancel{\oint \vec{E}\cdot da}}_{top} + \underbrace{\cancel{\oint \vec{E}\cdot da}}_{bottom} + \underbrace{\oint \vec{E}\cdot da}_{sides} = \vec{E}(\pi R^2)l$

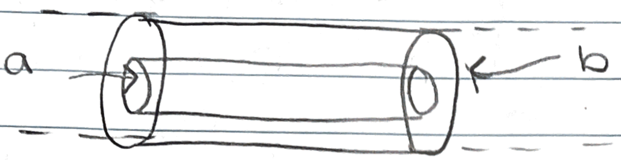
☆ slab, use GAUSS' LAW, center at origin ☆

$\boxed{1/27/23}$ GAUSS LAW WRAP-UP

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}$$

*ALWAYS true, but only useful w/cylindrical, sphereical, and planar

example: $\sigma > 0$ "nested cylinders"



→ find $\vec{E}$ everywhere:

$r > b$: outside all

$a < r < b$: in between

$r < a$: inside small

$\boxed{r<a}$: no e⁻-field due to $q_{encl} = 0$

$$\oint \vec{E}\cdot da = \int_{sides} E\cdot da \qquad da \neq 0 \rightarrow \vec{E} = 0$$

$\boxed{a<r<b}$: $q_{encl} = \underbrace{-\sigma 2\pi a l}_{\text{surface area small cylinder}}$

$$\oint \vec{E}\cdot d\vec{a} = \int E da = E\int da$$

$$= E 2\pi \cancel{r l}$$

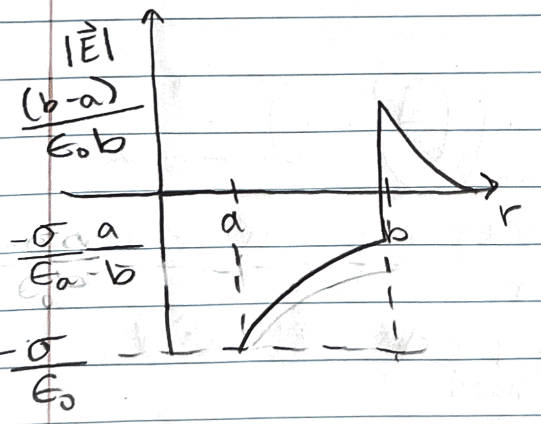
$$\rightarrow E = \boxed{\frac{-\sigma a}{r\epsilon_0}\hat{r}}$$

$\boxed{r>b}$: $q_{encl} = -\sigma 2\pi a l + \sigma 2\pi b l = \sigma(b-a) l 2\pi$

$$\oint \vec{E}\cdot d\vec{a} = \int E da = E 2\pi r l$$

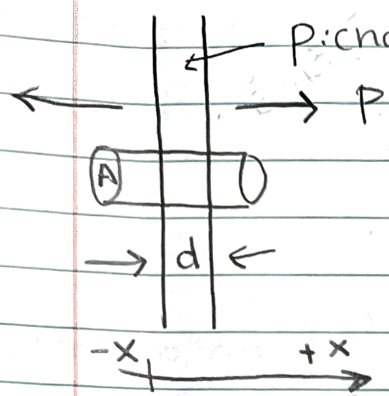
$$\boxed{\vec{E} = \frac{\sigma(b-a)}{\epsilon_0 r}\hat{r}}$$

plotting magnitude:



1/27/23 electrostatic pressure + energy in $\vec{E}$

- consider a thin slab



⇒ effective surface charge density: $\sigma = \rho d$

GAUSS SAYS: $\oint \vec{E}\cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}$

$$= E_1 A + E_2 A = \int \frac{\sigma dA}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

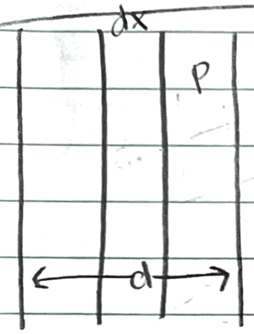
x-component across surface:

$$-E_1 + E_2 = \frac{\sigma}{\epsilon_0} \qquad \boxed{E_2 - E_1 = \frac{\sigma}{\epsilon_0}}$$

boundry condition

OR for a thin slab of thickness dx:

$$dE = \frac{\sigma}{\epsilon_0} = \frac{\rho dx}{\epsilon_0}$$



$$F = qE$$

$$dF = dq E$$

$$= dx\rho A E$$

$$\underbrace{\frac{dF}{A}}_{} = \rho dx E = \boxed{\epsilon_0 dE E}$$

bit of pressure $\left(\frac{\text{force}}{\text{area}}\right)$

★ The force can almost be written entirely as the electric field

electrostatic force

$$P_E = \frac{F}{A} = \frac{\int dF}{A} = \epsilon_0 \int_0^E E dE$$

↑from center

$$= \frac{\epsilon_0}{2} E^2$$

→ or from $x = -\frac{d}{2}$ to $x = \frac{d}{2}$:

$$\epsilon_0 \int_{E_1}^{E_2} E dE = \frac{\epsilon_0}{2}(E_2^2 - E_1^2)$$

→ across whole slab:

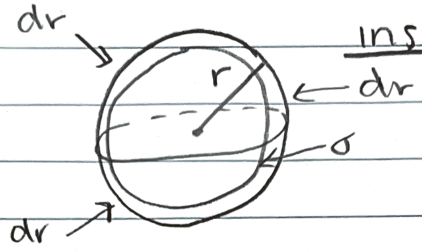
pressure: $p = \frac{\epsilon_0}{2}(E_2^2 - E_1^2)$

$$= \left(\frac{\epsilon_0}{2}\right)(E_2 - E_1)(E_2 + E_1)$$

$$= \underbrace{\epsilon_0 (E_2 - E_1)}_{\text{diff}} \underbrace{\left(\frac{E_2 + E_1}{2}\right)}_{\text{avg.}} = \boxed{\sigma E_{avg}}$$

→ looking at energy w/a sphere:

- hollow sphere w/surface charge density σ, radius r



inside: $E = 0$

- just outside:

$$E = \frac{\sigma}{\epsilon_0}$$

(using $E_2 - E_1 = \frac{\sigma}{\epsilon_0}$) → squish by 'dr' on all sides last time: electrostatic pressure 1/30/23

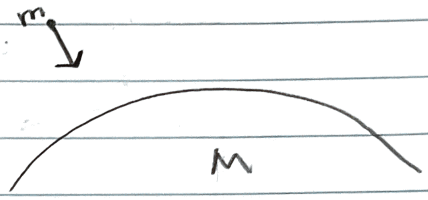
- energy density: $\frac{\epsilon_0 E^2}{2}$ (units $\frac{J}{m^3}$)

$\boxed{\text{electric potential}}$ (V)

→ to the familiar: $\boxed{\text{gravity}}$ $\vec{F}_g = \frac{-GmM}{r^2}\hat{r}$

energy associated w/gravity:

$$U = \frac{-GmM}{r}$$



$$\vec{g} = \frac{\vec{F}_g}{\textcircled{m}} = 9.8 m/s/s \text{ (on earth)}$$

charge on little mass

gravitational potential: $\Phi_g = \frac{U_g}{m} = -\frac{GM}{r}$

found by: how much work does it take to assemble charge distrib.

→ for the $\boxed{\text{electric field}}$

$$\vec{F}_c = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}$$

potential energy:

$$U_E = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r}$$

$$\vec{E} = \frac{\vec{F}_c}{q}$$

↔ how do we go btwn these?

electric potential: $V = \frac{U_E}{q} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$

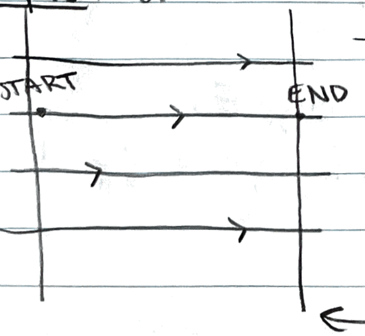
UNITS : $[V] = J/C =$ volts

work: $W = \underbrace{\Delta U}_{\text{change in potential from start to finish}} = -\int \vec{F}_1 \cdot d\vec{s}$

$$\boxed{\Delta V = \frac{\Delta U}{q} = -\int \vec{E} \cdot d\vec{s}}$$

#1: example: uniform $\vec{E}$ field (parallel so they don't grow in strength)

equipotentials are perpendicular to field lines



since $\vec{E}$ & $d\vec{s}$ in same direction,

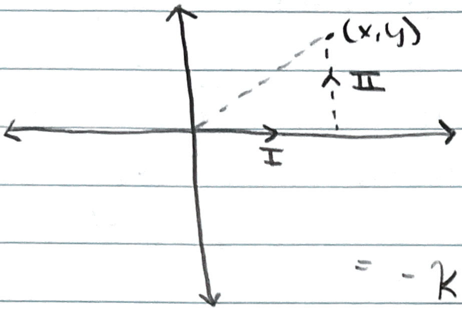
$$-\int \vec{E} \cdot ds = \int E ds = |\vec{E}| ds$$

← same potential along line

= $\boxed{\text{Equipotential surface}}$

(voltage decreasing from left to right)

#2: example: non-uniform e-field: $\vec{E} = K(y\hat{i} + x\hat{j})$



$$\Delta V = v(x,y) = -\int_{I} \vec{E}\cdot d\vec{s} - \int_{II} \vec{E}\cdot d\vec{s}$$

↑ If I assume/choice of $v(0)=0$

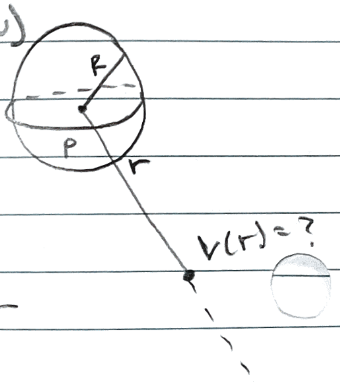
$$= -\int_0^x K(y\hat{i}+x\hat{j})\cdot dx\hat{i} - \int_0^y K(y\hat{i}+x\hat{j})\cdot\hat{j}dy$$

$$= -K\int_0^x \underset{\text{zero along path 1}}{y}\,dx - K\int_0^y x\,dy = Kx\int_0^y dy = \boxed{-Kxy}$$

$$\boxed{\text{potential} = v(x,y) = -Kxy}$$

#3: example: uniformly charged sphere

$\vec{E} = \frac{\rho R^3}{3\epsilon_0 r^2}\hat{r}$ (found using GAUSS' LAW)



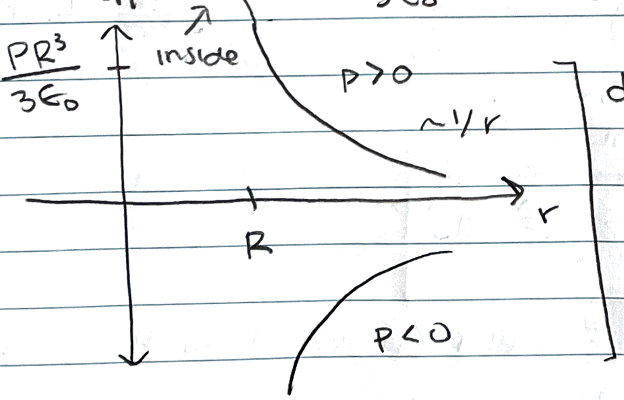
$$\Delta V = v(r) - \underset{\text{by choice} = 0}{v(\infty)} = v(r) = -\int_\infty^r \vec{E}\cdot d\vec{s}$$

$$= -\int_\infty^r \frac{\rho R^3}{3\epsilon_0 r^2}\hat{r}\cdot\hat{r}dr = \frac{-\rho R^3}{3\epsilon_0}\int_\infty^r \frac{1}{r^2}dr$$

limits encode direction ↗

$$= \frac{-\rho R^3}{3\epsilon_0}\left(\frac{-1}{r}\Big|_\infty^r\right) = \boxed{\frac{\rho R^3}{3\epsilon_0 r}} = \frac{QR^3}{4\pi\epsilon_0 r}$$

$v(\infty)=0$ ↑ "far away"



done in book!

→ Starting w/ V and finding E:

gradiant: $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ (an operator)

* note: little short path!

$$dV = -\vec{E}\cdot d\vec{s}$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}\partial z$$

$$= \vec{\nabla}V\cdot\underbrace{(\hat{i}dx + \hat{j}dy + \hat{k}dz)}_{d\vec{s}}$$

$$\vec{E} = -\vec{\nabla}V$$

last time: review

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electric potential: "V" or "ϕ"

$\boxed{\vec{E} = -\nabla V}$ using idea $\Delta V = -\int \vec{E} \cdot d\vec{s}$

$V = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{r}$, recalling for a single charge:

$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ (choosing $V=0$ at $r \to \infty$)

Similarly:

$\vec{F} = -\vec{\nabla} u$

$\underset{\vec{E}}{\frac{\vec{F}}{q}} = -\nabla \underset{V}{\frac{u}{q}}$

where:

$F_x = -\frac{dU}{dx}$

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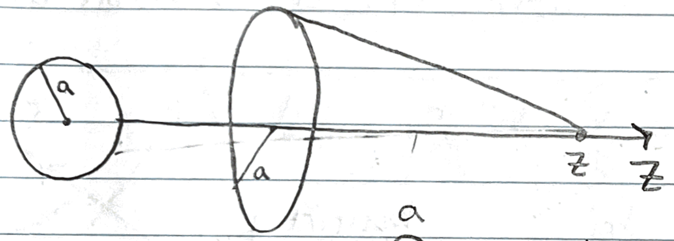
example: find V on axis of uniformly charged disk, σ charge density, radius of a.



$dQ = \sigma(2\pi r\, dr)$

$\mathcal{r} = \sqrt{z^2 + r^2}$

$V = \frac{1}{4\pi\epsilon_0}\int_0^a \frac{\sigma 2\pi r\, dr}{\sqrt{z^2+r^2}}$

$V = \frac{\sigma}{2\epsilon_0}\int_0^a \frac{r}{\sqrt{z^2+r^2}}\, dr$

→ what would be different w/electric field? we would have to deal w/ $\hat{r}$: $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dQ\hat{r}}{r^2}$

→ guess + check derivative:

guess: $\frac{\sigma}{2\epsilon_0}\left[\sqrt{z^2+r^2}\right]_0^a$

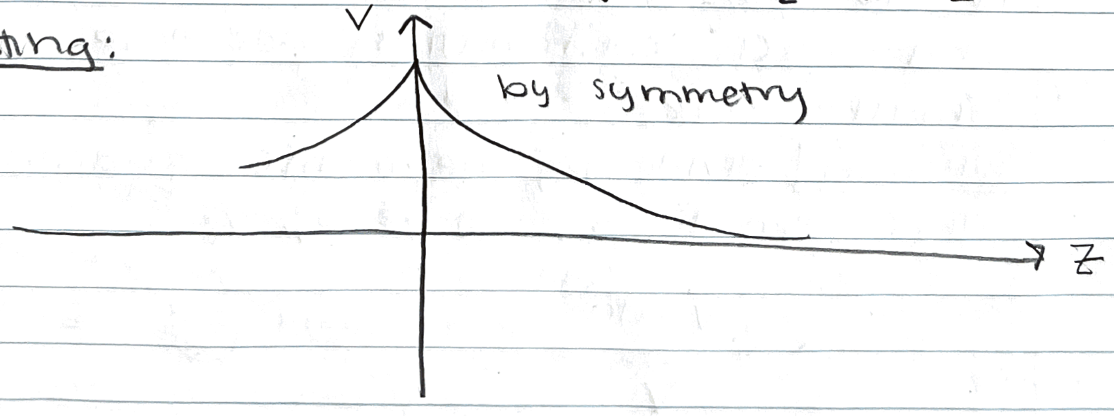
$= \left(\frac{\sigma}{2\epsilon_0}\right)\left(\sqrt{z^2+a^2} - z\right)$

check: $\frac{d}{dr}\sqrt{z^2+r^2} = \left(\frac{1}{2}(z^2+r^2)^{-1/2}\right.$

for $z > 0$

$\sqrt{z^2} = \pm z \to +z$

plotting:



by symmetry

Checking the previous example by: <u>taking limit of large z</u>!

$$V = \left(\frac{\sigma}{2\epsilon_0}\right)\left(z\underbrace{\sqrt{1+\frac{a^2}{z^2}}}_{\text{using taylor series: } (1+x)^n \simeq 1+nx} - z\right)$$

$$\simeq \left(\frac{\sigma}{2\epsilon_0}\right)\left(z\left(\cancel{1}+\tfrac{1}{2}\frac{a^2}{z^2}\right) - \cancel{z}\right)$$

$$= \frac{\sigma}{2\epsilon_0}\left(\frac{a^2}{2z}\right) = \frac{\sigma a^2}{4\epsilon_0}\left(\frac{1}{z}\right)\underbrace{\left(\frac{\pi}{\pi}\right)}_{} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{z}\right) \quad ☑$$

multiply by $\frac{\pi}{\pi}$ to get σa^2 to become Q

<u>WHAT IS $\vec{E}$ ALONG Z?</u>

$$\vec{E} = -\vec{\nabla}V$$

$$E_z = \frac{-dV}{dz} = \left(\frac{\sigma}{2\epsilon_0}\right)\left(\frac{z}{\sqrt{z^2+a^2}} - 1\right)$$

if $z=0$ $\longrightarrow$ $= \boxed{\frac{\sigma}{2\epsilon_0}}$ ☑ from GAUSS' LAW on an ∞ plane

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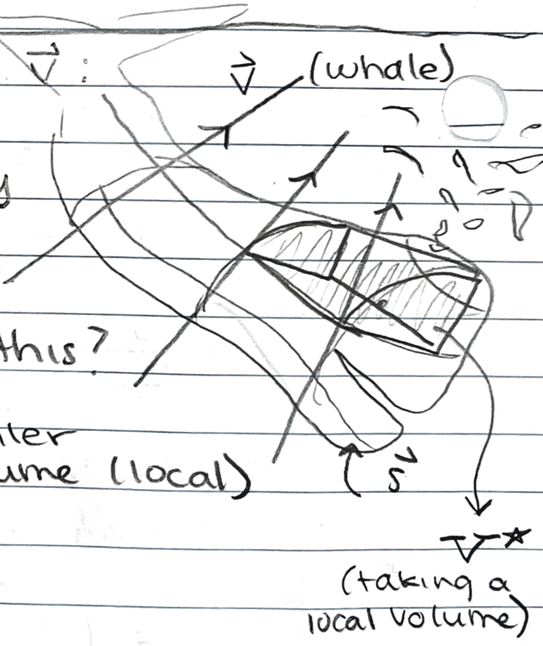
Theory <u>consider a vector field $\vec{V}$:</u>



$$\Phi_V = \oint_S \vec{V}\cdot d\vec{a}$$

boundry of $\mathcal{V}$: $\partial\mathcal{V}$

"GLOBAL" or "BOUNDRY"

→ Is there a <u>local</u> version of this?

$*\mathcal{V}$: $\Phi_i = \oint_{S_i} \vec{V}_1\, d\vec{a}$] flux of smaller chosen volume (local)

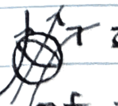
so: $$\Phi = \sum_{i=1}^{N} \int_{S_i} \vec{V}\cdot d\vec{a}$$

If $N\to\infty$, there would be <u>lots</u> of little boxes, and each contribution decreases to a vanishing small flux.

Solution divide by geometric quantity that also gets small as $N\to\infty$; V_i

$$\frac{\int_{S_i}\vec{V}\cdot d\vec{a}}{V_i}$$

: $\Phi = 0$



$\Phi \neq 0$

need <u>divergence</u> of field lines to get non-zero flux

divergence 2/1/23: cont.

def: $$\text{div}\,\vec V = \vec\nabla\cdot\vec V = \lim_{V_i\to 0}\frac{\int_{\partial V_i}\vec V\cdot d\vec a}{V_i}$$

→ DOES THIS EXIST?

(yes, but will not be proven now, wave hands + trust)

$\boxed{\nabla\cdot\vec V \text{ is our local version of "flux density"}}$

where $$\nabla\cdot\vec V \doteq \left(\hat\imath\frac{\partial}{\partial x}+\hat\jmath\frac{\partial}{\partial y}+\hat k\frac{\partial}{\partial z}\right)\left(V_x\hat\imath+V_y\hat\jmath+V_z\hat k\right)$$

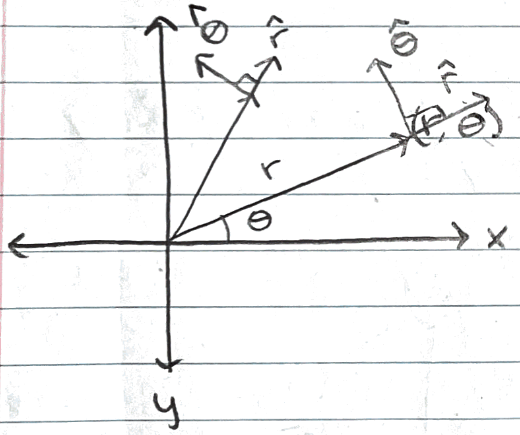
$$= V\frac{\partial}{\partial x}+V\frac{\partial}{\partial y}+V\frac{\partial}{\partial z}$$ (dot product removes most terms)

Theory continued $\nabla\cdot\vec E$ in cylindrical co-ord 2/3/23

* WHY DOES $\nabla\cdot\vec A$ IN CYLINDRICAL CO-ORDS LOOK WIERD?

(in appendix F) (r,θ,z)

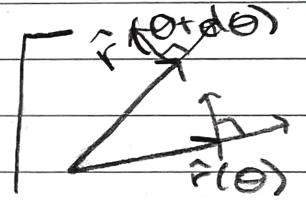
$$\vec\nabla\cdot\vec A = \left(\hat r\frac{\partial}{\partial r}+\hat\theta\frac{\partial}{\partial\theta}+z\frac{\partial}{\partial z}\right)\cdot\left(\hat r A_r+\hat\theta A_\theta+\hat z A_z\right)$$



unit vectors DEPEND on location

$$\frac{\partial\hat r}{\partial r}=0$$

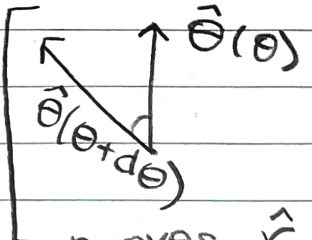
$$\frac{\partial\hat r}{\partial\theta}=\hat\theta \longrightarrow$$



move $\hat r$ in $\hat\theta$ direction

$$\frac{\partial\hat\theta}{\partial r}=0$$

$$\frac{\partial\hat\theta}{\partial\theta}=-\hat r \longrightarrow$$



moves $-\hat r$ as $\hat\theta$ changes

$$\vec\nabla\cdot\vec A = \overset{1}{\boxed{\cancel{\hat r\cdot\hat r}}}\frac{\partial A_r}{\partial r}+\hat\theta\,\overset{\hat\theta}{\boxed{\cancel{\frac{\partial\hat r}{r\partial\theta}}}}A_r+\overset{1}{\boxed{\cancel{\hat\theta\cdot\hat\theta}}}\frac{\partial A_\theta}{r\partial\theta}$$
$$+\underbrace{\cancel{\hat z\cdot\hat z}}_{1\quad 1}\frac{\partial A_z}{\partial z}+\hat\theta\,\underbrace{\cancel{\frac{\partial\hat\theta}{r\partial\theta}}}_{-\hat r}A_\theta$$

$$=\frac{\partial A_r}{\partial r}+\underbrace{\cancel{\hat\theta\hat\theta}}_{1}A_r+\underbrace{\cancel{\hat\theta\cdot\hat\theta}}_{1}\frac{\partial A_\theta}{r\partial\theta}+\frac{\partial A_z}{\partial z}+\underbrace{\cancel{\hat\theta(-\hat r)A_\theta}}_{0}$$

$$=\frac{\partial A_r}{\partial r}+\frac{A_r}{r}+\frac{\partial A_\theta}{r\partial\theta}+\frac{\partial A_z}{\partial z} \rightarrow \frac{A_r}{r}=\frac{\partial r}{\partial r}\frac{1}{r}A_r$$

$$=\boxed{\frac{1}{r}\frac{\partial}{\partial r}(rA_r)+\frac{1}{r}\frac{\partial A_\theta}{\partial\theta}+\frac{\partial A_z}{\partial z}}$$

(! draw pictures to help w/confusion)

2/3/23

$$\nabla\cdot E = \lim_{V_i\to 0}\frac{1}{V_i}\int_{\partial V_1} E\text{-}da_i$$

Immediate consequence: **DIVERGENCE THEOREM**

L-H side of GAUSS LAW

$$\Phi_E = \boxed{\int_{\partial V}\vec E\cdot d\vec a} = \lim_{N\to\infty}\sum_{i=1}^{N}\int_{S_i}\vec E\cdot da_i\left(\frac{V_i}{V_i}\right)$$

$$= \underbrace{\lim_{N\to\infty}\sum_{i=1}^{N}\underbrace{\frac{1}{V_i}\int_{S_i}\vec E da_i}_{\vec\nabla\cdot\vec E}\, V_i}_{\text{volume integration}} = \boxed{\int_V \vec\nabla\cdot\vec E dv}$$

divergence theorem: $\int_V \vec\nabla\cdot\vec A dV = \int_{\partial V}\vec A\cdot d\vec a$

also! the flux is in GL:

$$\oint_{\partial V}\vec E\cdot d\vec a = \frac{q_{encl}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_V \rho dv = \int_V \vec\nabla\cdot\vec E dv$$

$$\Rightarrow \int_V\left(\vec\nabla\cdot\vec E - \rho/\epsilon_0\right)dV = 0 \quad \text{for ALL volumes}$$

another form of GAUSS' LAW $\boxed{\vec\nabla\cdot\vec E = \frac{\rho}{\epsilon_0}}$ holds everywhere in space (x,y,z)

Summarizing diagram:

! will be most usefull !

$$\begin{tikzcd}
 & \rho \arrow[dl, bend right, "\vec E = \frac{1}{4\pi\epsilon_0}\int\frac{\rho\hat r}{r^2}"'] \arrow[dr, bend left, "V=\frac{1}{4\pi\epsilon_0}\int\rho\frac{dV}{r}"] & \\
\vec E \arrow[ur, "\rho=\epsilon_0\vec\nabla\cdot\vec E"] \arrow[rr, "V=-\int\vec E\cdot d\vec s"] & & V \text{ or } \phi \arrow[ul, "\boxed{\rho=-\epsilon_0\nabla^2 V}"'] \arrow[ll, bend left, "\vec E=-\nabla V"]
\end{tikzcd}$$