E and B transformations w/ $\vec{v} = v\hat{i}$: 4/17/23

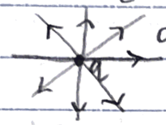
$$\begin{cases} E'_x = E_x \\ E'_y = \gamma(E_y - vB_z) \\ E'_z = \gamma(E_z + vB_y) \end{cases}$$

$$\begin{cases} B'_x = B_x \\ B'_y = \gamma(B_y + vE_z/c^2) \\ B'_z = \gamma(B_z - vE_y/c^2) \end{cases}$$

Today: example of transformations + start on dynamics

① in frame S, there is a single point charge, q

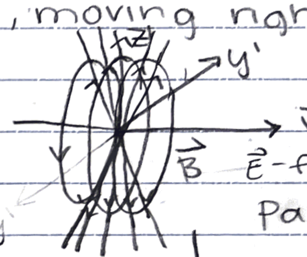
in S: $\vec{E}$



$\vec{B} = 0$ (no moving charge)

charge moving

in S', moving right at $\vec{v}$:



$\vec{E}$-field gets pancaked

★ in S', at constant $r = \sqrt{x^2 + y^2}$, E_z and E_y are the same magnitude ★

$\vec{B}'_x = 0$

$\vec{B}'_y = \gamma\left(\frac{vE_z}{c^2}\right)$

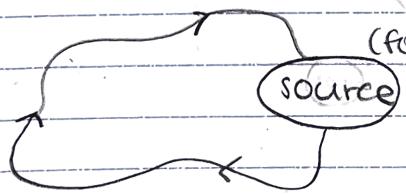
$\vec{B}'_z = \gamma\left(\frac{-vE_y}{c^2}\right)$

$\vec{B}' = \vec{v} \times \vec{E}$

$\vec{B}$-field circulating around moving charge on x-axis

② current-carrying wire in simple circuit

Charge is placed inside of the wire

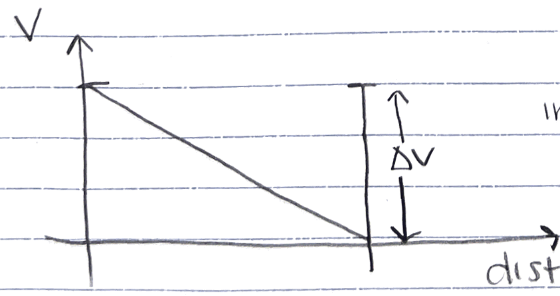


(for instance, a battery)

local force per unit charge:

$$\frac{\vec{F}}{q} = \vec{f} = \vec{f}_{source} + \vec{E}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$



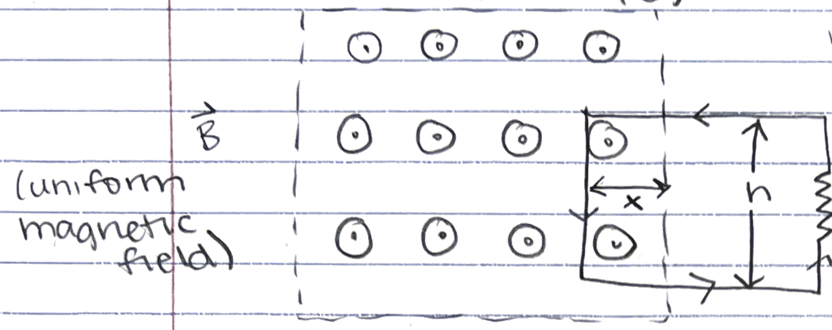
in ideal case:

$$\oint \vec{f} \cdot d\vec{s} = 0 = \oint \vec{f}_{source} \cdot d\vec{s} + \oint \vec{E} \cdot d\vec{s}$$

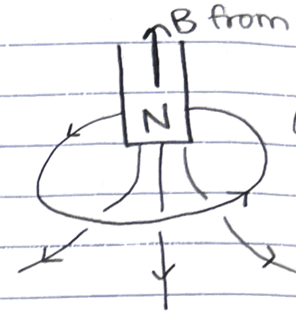
so: $\underbrace{\oint \vec{f}_{source} \cdot d\vec{s}}_{\text{this drives ~~the current~~}} = \underbrace{-\oint \vec{E} \cdot d\vec{s}}_{\text{potential}}$

$$\boxed{\begin{array}{c} \mathcal{E} = \oint f_{source} \cdot d\vec{s} \\ \text{EMF} = \text{electro-negative force} \\ \text{(force per unit charge)} \end{array}}$$

③ consider the configuration:
(in real life)



$V = \frac{dx}{dt}$



LENS SIGN

$f_{mag} \neq 0$

$\vec{F}_{mag} = q\vec{V} \times \vec{B}$

between a and b:

$F_{mag} = qVB$ (down)

$f_{mag} = VB$

So, $\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{s}$

$= VBh$

⇒ how is this related more precisely to $\vec{B}$?

consider: $\Phi_B = \int \vec{B} \cdot d\vec{a} = Bhx$

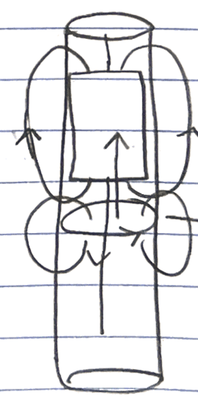
★ some relation:

$$\boxed{\mathcal{E} = \frac{-d\Phi_B}{dt}}$$

nature abhors a change in flux

$$\frac{d\Phi_B}{dt} = Bh\underbrace{\frac{dx}{dt}}_{\text{decreasing}} = VhB = \mathcal{E}$$

→ passing a cylindrical magnent through a metal tube:



(eady current) → current inducing mag[illegible]tic fie[illegible] in oppos[illegible]ed d[illegible]ection last time: EMF (electro-motive-force) 4/19/23

↳ really a potential!

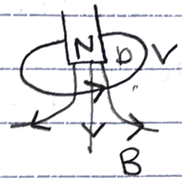
$$\mathcal{E} = \oint_c \frac{F_c}{q} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = \frac{-d}{dt} \int_s \vec{B} \cdot d\vec{a}$$

↖ same dimensions as E

$$\mathcal{E} = \frac{-d\Phi_B}{dt}$$

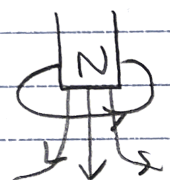
LENZ'S SIGN

3 classic experiments:

I) 

magnent through a loop of current

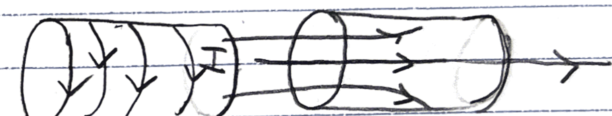
II) (w/ SB same as example I)



(loop moving relative to the magnent instead)

$\frac{dI}{dt} > 0$

III) changing B:



turn on current through solenoid, producing flux through the secound solenoid

$\vec{B}$ to right is increase

$\boxed{\text{LENZ'S SIGN}}$ wants to oppose the direction of flux

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There is an (induced) electric field, E, from the change in flux. (single charge)

$$\boxed{\mathcal{E} = \oint_c \vec{E} \cdot d\vec{s} = \frac{-d}{dt} \int \vec{B} \cdot d\vec{a}}$$ FARADAY'S LAW

differential form: stoke's theorem: $\oint_{\partial S} \vec{v} \cdot d\vec{s} = \int_S \vec{\nabla} \times \vec{v} \cdot d\vec{a}$

$$\oint_c \vec{E} \cdot d\vec{s} = \int_S \vec{\nabla} \times \vec{E} \cdot d\vec{a} = \frac{-d}{dt} \int \vec{B} \cdot d\vec{a}$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}}$$ differential form of faraday's law

story so far: 4/19/23

① $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ ~~(GAUSS~~ GAUSS' LAW

② $\vec{\nabla}\cdot\vec{B} = 0$ (no magnetic charges, monopole)

③ $\vec{\nabla}\times\vec{E} = \frac{-\partial B}{\partial t}$ FARADAY'S LAS

④ $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ AMPERE'S LAW

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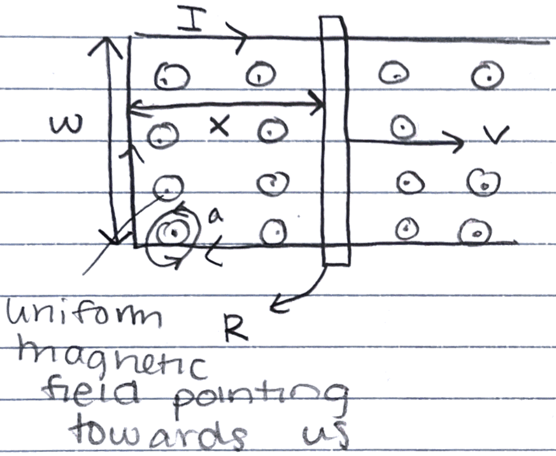
example: (faraday's)



$\mathcal{E} = IR = \frac{-d}{dt}\int \vec{B}\cdot d\vec{a}$

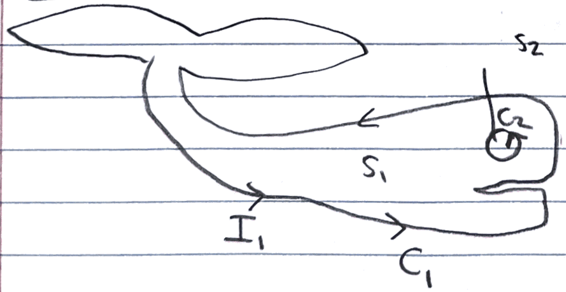
$\frac{d}{dt}\int B\,da = \frac{-d}{dt}B\underbrace{\int da}_{wx}$

$= \frac{-d}{dt}Bwx = \boxed{-Bwv}$

current runs clockwise as anticipated

$\boxed{I = \frac{-Bwv}{R}}$

inductance (self, mutual, ect.)



current-carry whale, with a current carrying eye enclosed

⇒ what is the affect on loop C_2 of driving I_1 around C_1?

$\mathcal{E}_2 = \frac{-d}{dt}\Phi_{21} = \frac{-d}{dt}\underbrace{\int_{S_2} B_1\cdot da_2}_{\propto I_1 M_{21}}$

$= -M_{21}\frac{dI_1}{dt}$

mutual inductance

units:

$[M] = \frac{Vs}{A} = \Omega\cdot S$

$1H = \frac{1Vs}{A}$ (Henry)

(driven other direction cont. on next page)

4/19/23

$$\mathcal{E}_1 = \frac{-d\Phi_{12}}{dt} = -M_{12}\frac{dI_2}{dt}$$

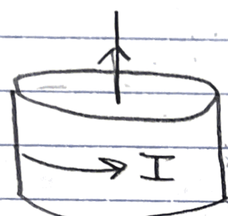
⇒ how are M_{12} and M_{21} related? (they are equal)

Last time: Faraday's Law: 4/21/23

$$\underbrace{\int \vec{E}\cdot d\vec{s}}_{\mathcal{E}} = \frac{-d}{dt}\underbrace{\int \vec{B}\cdot d\vec{a}}_{\Phi_B}$$

Today: Thompson jumping rings: $I(t) = I_0\cos(\omega t)$

$f = 60\,\text{Hz}$



increasing

→ current causes ring to jump

w/ ring + slice → doesn't jump

w/ copper ring → heavier but better flux = same sized jump as before

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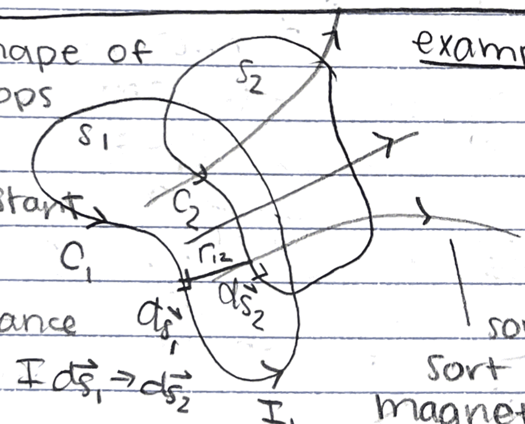

shape of loops is constant

r_{12}: distance btween $I\,d\vec{s}_1 \to d\vec{s}_2$

some sort of magnetic flux activity

example: EMF in two loops

$$\mathcal{E}_2 = \frac{-d}{dt}\int_{S_2} B_1\cdot d\vec{a}_2$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

under certain conditions:

$$\nabla^2\vec{A} = \mu_0\vec{J} \quad \left(\nabla^2 V = \frac{\rho}{\epsilon_0}\right)$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{s}}{r}$$

(by analogy)

$$\mathcal{E}_2 = \frac{-d}{dt}\int\vec{\nabla}\times\vec{A}_1\cdot d\vec{a}_2$$

stoke's theorem

$$= \frac{-d}{dt}\int_{C_2}\vec{A}_1\cdot d\vec{s}_2 = \frac{-d}{dt}\int_{C_2}\int_{C_1}\frac{\mu_0}{4\pi}\frac{I_1\,d\vec{s}_1\cdot d\vec{s}_2}{r_{12}}$$

current has no time dependance: shape of loops = constant

now $\vec{A}_1 = \frac{\mu_0}{4\pi}\int_{C_1}\frac{I_1\,d\vec{s}_1}{r_{12}}$

$$= \frac{-dI_1}{dt}\underbrace{\frac{\mu_0}{4\pi}\int_{C_1}\int_{C_2}\frac{d\vec{s}_1\cdot d\vec{s}_2}{r_{12}}}_{}$$

geometry only: depends on shape = mutual inductance: M_{12}

★continued on next page★

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$$\mathcal{E}_2 = -M_{12}\frac{dI_1}{dt}$$

⇒ now in terms of effect on other ring(1):

$$\mathcal{E}_1 = \frac{-d}{dt}\int_{S_1} B_2 \cdot d\vec{a}_1 = \frac{dI_2}{dt}\underbrace{\frac{\mu_0}{4\pi}\int_{C_1}\int_{C_2}\frac{d\vec{s}_2 \cdot d\vec{s}_1}{r_{21}}}_{M_{21} = M_{12}}$$

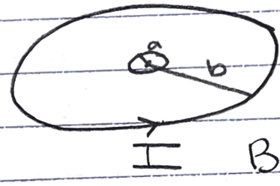
There fore: $M = M_{12} = M_{21}$

→ doesn't matter!

given a certain configuration we can compute M, using soley geometry: find flux + divide by I

example:

(little loop: rad a inside of a big loop: rad b)



magnetic field on axis of a loop is:

$$B_z = \frac{\mu_0 I b^2}{2(b^2+z^2)^{3/2}}$$

in loop a: a<<b

→ if there are concentric circles: $B \approx \frac{\mu_0 I}{2b}$

$$\Phi = \int \vec{B}\cdot d\vec{a} = B\pi a^2$$ since $B \approx$ constant in loop a

$$B\pi a^2 = \frac{\mu_0 \pi I a^2}{2b}$$

$$M = \frac{\Phi}{I} = \frac{\mu_0 \pi \cancel{I} a^2}{2b\cancel{I}} = \boxed{\frac{\mu_0 \pi a^2}{2b}}$$

→ suppose instead of single loop a, "n" loops!

$$M = N\left(\frac{\mu_0 \pi a^2}{2b}\right)$$

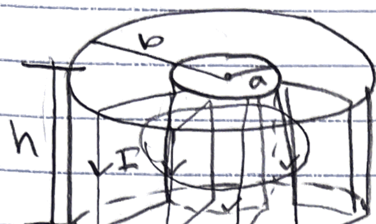
This works for one thing, too

⇒ self inductance: L

$$\mathcal{E} = \frac{-dI}{dt}L$$

4/21/23

example: toroid w/ N windings:



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

amperean loop = circle around a w/radius "r"

$$B\,2\pi r = \mu_0 I_{encl} = \mu_0 I N$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}\,\hat{\phi}}$$

flux: $\Phi = \int \vec{B}\cdot d\vec{a}$

$= \int B\,dy\,dr = \int dy \int B\,dr = h\int B\,dr$

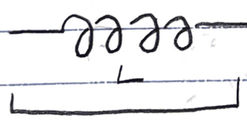
$= \frac{h\mu_0 N I}{2\pi}\int_a^b \frac{1}{r}dr = \frac{h\mu_0 NI}{2\pi}\ln(r)\Big|_a^b$

$= \frac{h\mu_0 NI}{2\pi}\ln\left(\frac{b}{a}\right)$ for 1 loop

→ for all loops: $\Phi = N\left(\frac{h\mu_0 NI}{2\pi}\ln\left(\frac{b}{a}\right)\right) = \frac{h\mu_0 N^2 I}{2\pi}\ln\left(\frac{b}{a}\right)$

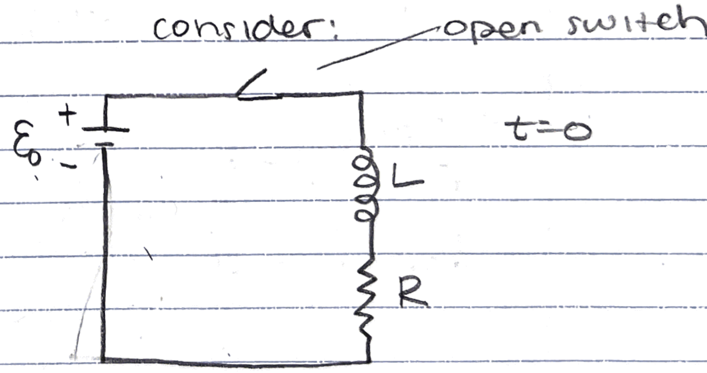
Therefore $L = \frac{\Phi}{I} = \frac{\mu_0 N^2 I h}{2\pi I}\ln\left(\frac{b}{a}\right) = \boxed{\frac{\mu_0 N^2 h \ln\left(\frac{b}{a}\right)}{2\pi}}$

In circuits, inductors are denoted

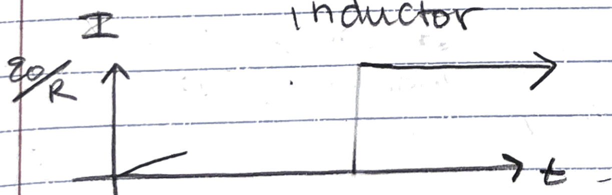


inductor

consider:



t=0



→ closing switch, t>0:

$I(t) =$

kirkoff loop rule:

$$\mathcal{E}_0 - L\frac{dI}{dt} - IR = 0$$

→ use diff eq:

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{\mathcal{E}_0}{L}$$