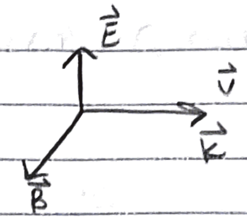
last time: plane and E&M waves:  5/1/23

$\vec{E}(t,\vec{r}) = \vec{E}_0 \cos(\vec{k}\vec{r} \pm \omega t)$

$\vec{B}(t,\vec{r}) = \vec{B}_0 \cos(\underbrace{\vec{k}\vec{r}}_{[k_x x + k_y y + k_z z]} \pm \omega t)$

$\vec{V}_1 \times \vec{E}_0 = \vec{B}_0$

$|\vec{B}_0| = \frac{|\vec{E}_0|}{c}$

today:

*final talk: quantum gravity → in class on Friday?
→ 4:10pm talk on Thurs

$\boxed{\text{poynting vector}}$ (energy in waves)

$u = u_E + u_B$ [consider energy density]

$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$

in plane waves: $u = \frac{1}{2}\epsilon_0 |\vec{E}_0|^2 = \frac{1}{2}\frac{1}{\mu_0}|\vec{B}_0|^2 \cos^2(\ldots)$

$(\ldots) = \vec{k}\cdot\vec{r} \pm \omega t$

$|\vec{B}_0|^2 = \frac{E_0^2}{c^2} = \frac{E_0^2}{\mu_0\epsilon_0}$

$= \epsilon_0 E_0^2 \cos^2(\ldots)$

average energy density:

$$\bar{u} = \frac{1}{T}\int_0^T u\,dt = \frac{1}{2}\epsilon_0 E_0^2$$

[? connection: $E = hf$, $E = \omega\hbar$?]

change in u is:

$$\frac{\partial u}{\partial t} = \frac{\epsilon_0}{\cancel{2}}\left(\cancel{2}\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}\right) + \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$$

⇒ maxwell's equations in vacuum:

$\vec{\nabla}\cdot\vec{E} = 0$ $\qquad$ $\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$ $\qquad$ $\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

$$= \cancel{\epsilon_0}\vec{E}\cdot\frac{\vec{\nabla}\times\vec{B}}{\mu_0\cancel{\epsilon_0}} = \frac{\vec{E}\cdot\vec{\nabla}\times\vec{B}}{\mu_0} + \frac{1}{\mu_0}\vec{B}\cdot(-\vec{\nabla}\times\vec{E})$$

$$= \frac{\vec{E}\cdot(\vec{\nabla}\times\vec{B}) - \vec{B}\cdot(\vec{\nabla}\times\vec{E})}{\mu_0}$$

*recall: $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$ *

$$= \boxed{\frac{\vec{\nabla}\cdot(\vec{B}\times\vec{E})}{\mu_0}} = \boxed{\frac{-\vec{\nabla}\cdot(\vec{E}\times\vec{B})}{\mu_0}}$$

for wave, $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \parallel \vec{k}$

(parallel to $\vec{k}$)

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$: poynting vector

→cont. on back

5/1/23

$\boxed{\frac{\partial u}{\partial t} + \nabla \cdot \vec{S} = 0}$ OR comp. w/: $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

↳ energy neither created nor destroyed
= version of conservation of energy

↳ charge neither created nor destroyed

units of $[\vec{S}] = \frac{[E][B]}{[\mu_0]}$ (square brackets to represent units)

$= \frac{N}{C}(T)\frac{C^2}{kg \cdot m} = \left(\frac{N}{C}\right)\left(\frac{C^2}{kgm}\right)\left(\frac{kg}{Cs}\right) = \frac{N}{m \cdot s} = \frac{kg\,m}{s^2 \cdot ms}$

$[\vec{S}] = \frac{kg}{s^3}$

power per area: Intensity

power in watts $= \frac{kgm^2}{s^2}\left(\frac{1}{s}\right) \to \frac{kgm^2}{s^3}\frac{1}{m^2} = \boxed{\frac{W}{m^2}}$

Intensity: I : magnitude of $\vec{S}$ (poynting vector)

$I = |\vec{S}|$ ↳ $\vec{S}$ points in direction of energy flow

$I_0 = 1360$ watts/m^2 (sunlight's intensity at top of atmosphere)

time average

- If there is mechanical work, W:

e&m conservation of energy: $\frac{dW}{dt} = \frac{\partial \int_V u\,dV}{dt} - \int_V \vec{\nabla}\cdot\vec{S}\,dV = \frac{-d}{dt}\int_V u\,dV - \oint_{\partial V} \vec{S}\cdot d\vec{a}$

→ for plane waves:

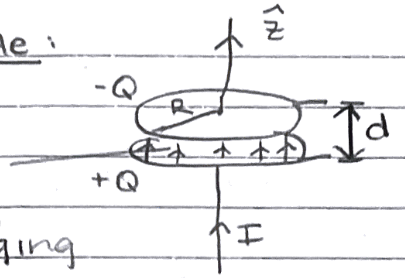
$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}\frac{E_0^2}{c}\cos^2(\cdots)\hat{k}$ ← direction

$= \sqrt{\mu_0 \epsilon_0}\,E_0^2\cos^2(\cdots)\hat{k}$

$= c\epsilon_0 E_0^2\cos^2(\cdots)\hat{k}$ → taking the time average

$|\vec{S}| = \underbrace{\frac{1}{2}\epsilon_0 c E_0^2}_{I\,=\,\text{intensity}}\hat{k}$

example: 5/1/23



e-field lines ignoring the fringing field

disk capacitor w/seperation "d", radius "R"
★constant charge, I ★

E-field: $\frac{\sigma}{\epsilon_0}$ $\hat{z}$

$\vec{E} = \frac{Q}{\epsilon_0 A}\hat{z}$

$\rightarrow$ flux of E is growing: $\int \vec{E}\cdot d\vec{a}$ is increasing

$-\frac{d}{dt}\int \vec{E}\cdot d\vec{a} = \int \vec{B}\cdot d\vec{s}$; what is relationship to the magnetic field?

last time: poynting vector: ★monday = review★ 5/3/23

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

Today: above example continued:

- let's find the B-field

$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$

$\oint \vec{B}\cdot d\vec{s} = \mu_0\epsilon_0\frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{a}$

right-hand side

$\mu_0\epsilon_0\frac{\partial}{\partial t}\int_S \vec{E}\cdot d\vec{a} = \mu_0\epsilon_0\frac{d}{dt}E\int da$

$= \mu_0\epsilon_0\frac{d}{dt}\left(\frac{Q}{\epsilon_0 \cancel{\pi r^2}}\right)\cancel{\pi r^2} = \mu_0\frac{\cancel{\epsilon_0}}{\cancel{\epsilon_0}}\frac{dQ}{dt} = \boxed{\mu_0 I}$

$= \frac{1}{c^2}\frac{dE}{dt}\pi r^2$

left-hand side

$\oint_{\partial S} \vec{B}\cdot d\vec{s} = B(2\pi r)$

$\Rightarrow B(2\cancel{\pi}\cancel{r}) = \frac{1}{c^2}\frac{dE}{dt}\cancel{\pi}\cancel{r}^2$

$2B = \frac{1}{c^2}\frac{dE}{dt}r$

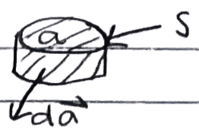
$\boxed{\vec{B} = \frac{r}{2c^2}\frac{dE}{dt}\hat{\phi}}$

$\vec{E}\times\vec{B} = |\vec{E}|\frac{r}{2c^2}\frac{dE}{dt}(\hat{z}\times\hat{\phi})$

poynting vector inside the two plates

$\rightarrow \vec{S} = \frac{1}{\mu_0}\left(|\vec{E}|\frac{dE}{dt}\frac{r}{2c^2}\right)(-\hat{r}) = \boxed{\frac{1}{2}\epsilon_0 E\frac{dE}{dt}(-\hat{r})}$

verify that $\underbrace{\int \vec{S}\cdot d\vec{a}} + \frac{d}{dt}\int u\, dV = 0$



(pointing out)

$\int_S \vec{S}\cdot d\vec{a} = \int_S \frac{1}{2}\epsilon_0 E\frac{dE}{dt}(-1)(a)(2\pi a\cdot d)$

$\Rightarrow$ continued on back

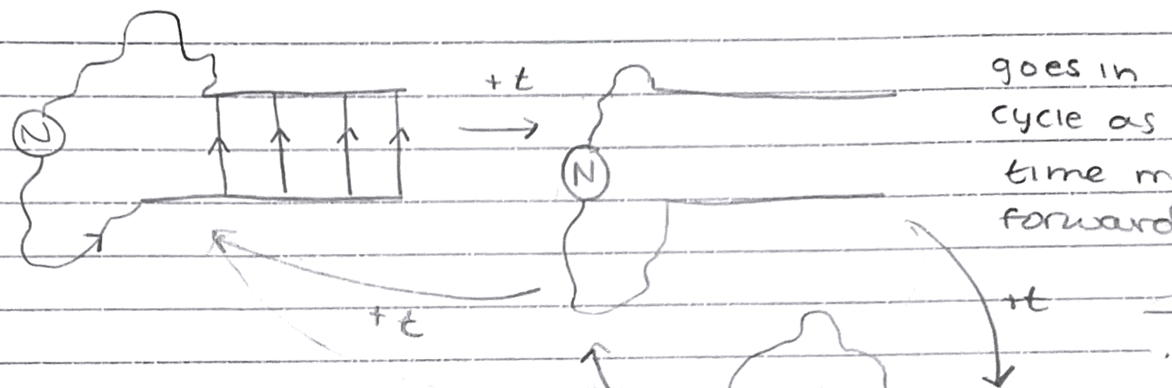
while: $\frac{d}{dt}\int \frac{1}{2}\epsilon_0 E^2 dV = \frac{d}{dt}\left(\frac{1}{2}\epsilon_0 E^2\right)\left(\overbrace{\int dv}^{\pi a^2 d}\right) = \boxed{\frac{1}{2}\epsilon_0 2E\frac{dE}{dt}\pi a^2 d}$

(from previous): $\int_S \vec{S}\cdot d\vec{a} = \boxed{-\epsilon_0 E\frac{dE}{dt}\pi a^2 d}$

$$\frac{1}{2}\epsilon_0 \frac{EdE}{dt}\pi a^2 d \longleftrightarrow -\epsilon_0 E\frac{dE}{dt}\pi a^2 d$$

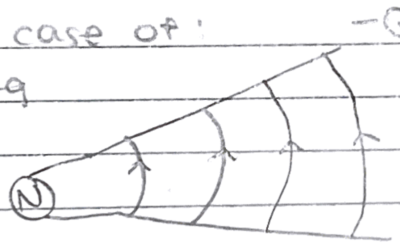
Therefore: $\int \vec{S}\cdot d\vec{a} + \frac{d}{dt}\int u\, dv = 0$

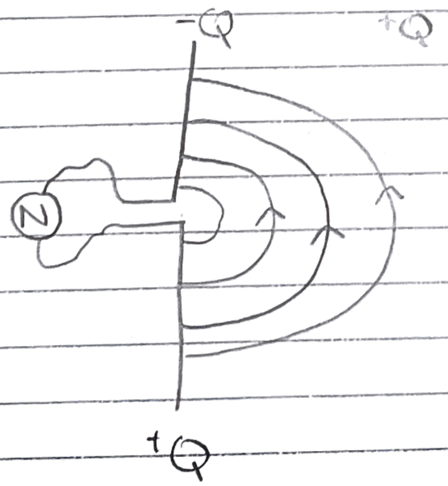
<u>DEMOS</u>: antena and polarization

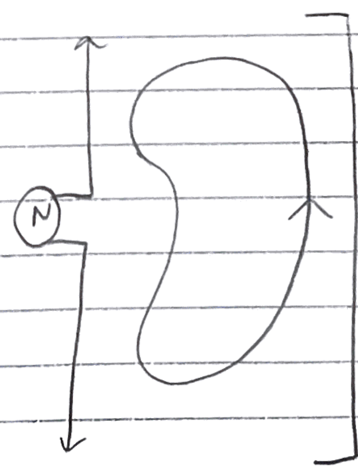


goes in cycle as time moves forwards

In case of: (pulling plates apart)







(at $v=c$) w/oscillating plates, changing charge

→ shown in live demo w/light bulb

observation from 16 year old Einstein: 5/3/2w

⇒ what is light like if you could catch up with it?

-change frame: $\vec{V} = v\hat{i}$

↳ transformation equations:

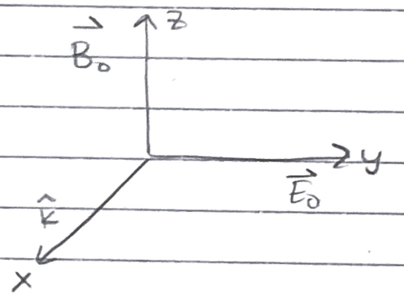
$$E_x' = E_x$$
$$E_y' = \gamma(E_y - vB_z)$$
$$E_z' = \gamma(E_z + vB_y)$$

$$B_x' = B_x$$
$$B_y' = \gamma\left(B_y + \frac{v}{c^2}E_z\right)$$
$$B_z' = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

$$|\vec{B}| = \frac{|\vec{E}_0|}{c}$$
$$\vec{E} = E_0\cos(kz - \omega t)\,\hat{y}$$
$$\vec{B} = \frac{E_0}{c}\cos(kz - \omega t)\,\hat{z}$$

$$E_y' = \gamma(E_y - vB_z)$$
$$= \gamma\left(E_0 - \frac{v}{c}E_0\right)$$
$$= \gamma E_0\left(1 - \frac{v}{c}\right)$$
$$= \frac{\left(1 - \frac{v}{c}\right)}{\sqrt{\left(1-\frac{v}{c}\right)\left(1-\frac{v}{c}\right)}}E_0 = \overbrace{\sqrt{\frac{1-\frac{v}{c}}{1+v/c}}}^{\text{K-factor/red shift}}\,E_0$$

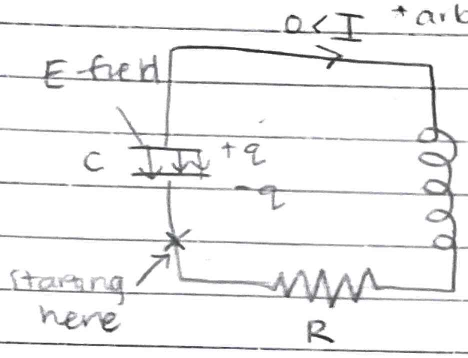
as $v \to c$: waves fade to zero



last time: waves and poynting vector 5/5/23

$\Rightarrow$ today: LRC circuits

0 < I *arbitrary choice*



$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$

(width = d)

across the capacitor:

$\Delta V = -\int \vec{E} \cdot d\vec{s} = E \int d\vec{s}$

$= \boxed{\frac{qd}{\epsilon_0 A} = \frac{q}{C}}$ positive given the choices we made

C = capacitance ≠ speed of light

$+\frac{q}{C} - L\frac{dI}{dt} - IR = 0$

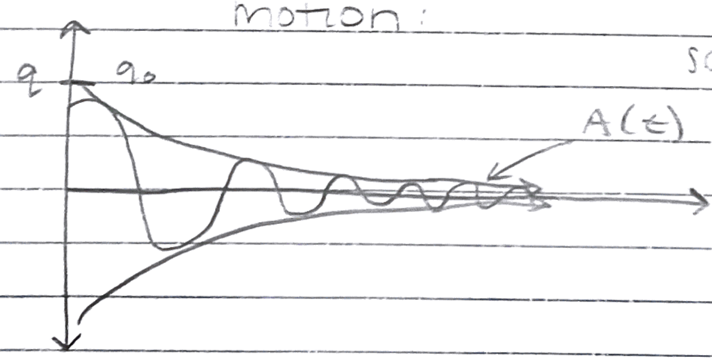
→ given the direction of $I > 0$, $I = -\frac{dq}{dt}$

$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$ (oscillatory motion)

$\frac{d^2x}{dt^2} + \underbrace{\boxed{\frac{b}{m}}}_{2\beta}\frac{dx}{dt} + \underbrace{\boxed{\frac{k}{m}}}_{\omega_0^2}x = 0$

$\Rightarrow$ damped harmonic motion:

solution: $q(t) = q_0 e^{-\beta t}\sin(\omega t + \phi)$



$\beta_{elect} = \frac{R}{2L}$

$\omega = \sqrt{\omega_0^2 - \beta^2}$ from pluggin in trial solution into equation

electrical	vs.	mechanical
q		x
I		$V = \frac{dx}{dt}$
L		m
b		R
C		$1/k$

(lim as $R \to 0$ on next page)

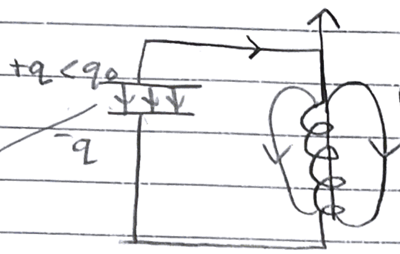
5/5/23 ·15·

In the limit as $R \to 0$: $\beta \to 0$, $\omega \to \omega_0 = \frac{1}{\sqrt{LC}}$

$q(t) \to q_0 \sin(\omega_0 t + \phi)$

In case of $t \to T$

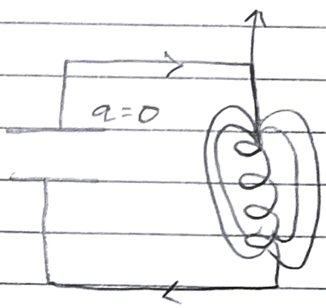
(☆ excluding resistor b/c assuming small)



B-field: w/ energy density $\frac{1}{2}\frac{B^2}{\mu_0}$

e-field w/ energy density: $\frac{1}{2}\epsilon_0 E^2$

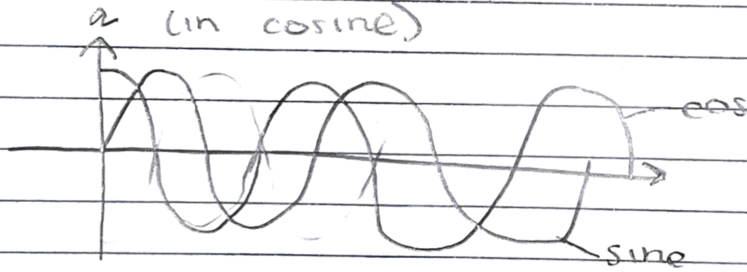
$t > T$



B-field (now greater)

Cycles through having a large e-field and then a large b-field and so forth:

q (in cosine)



$I = -\frac{dq}{dt}$

LC - circuit

→ suppose we add an oscillating voltage: then equation of motion becomes:

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = \frac{\mathcal{E}_0}{L}\cos(\omega t)$$

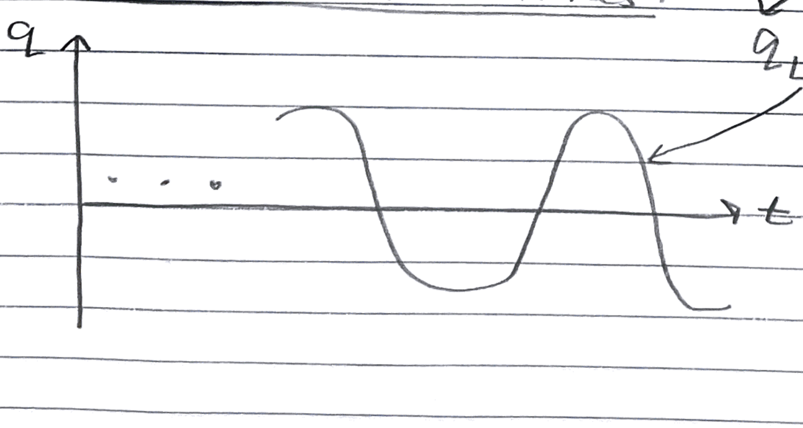
↳ driving frequency

Solution at late times:

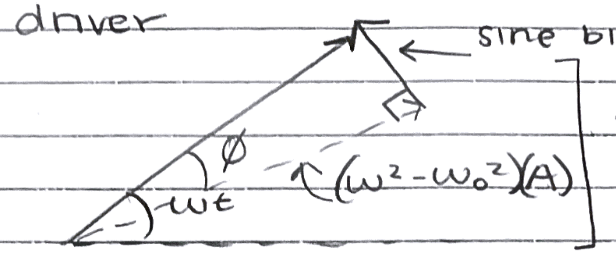
late time q

$q_L(t) = A(\omega)\cos(\omega t - \phi)$

↳ amplitude is constant in time, all peaks are the same



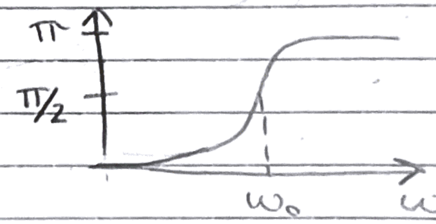
phasor method to solve for $A(\omega)$ and ϕ: 5/5/23



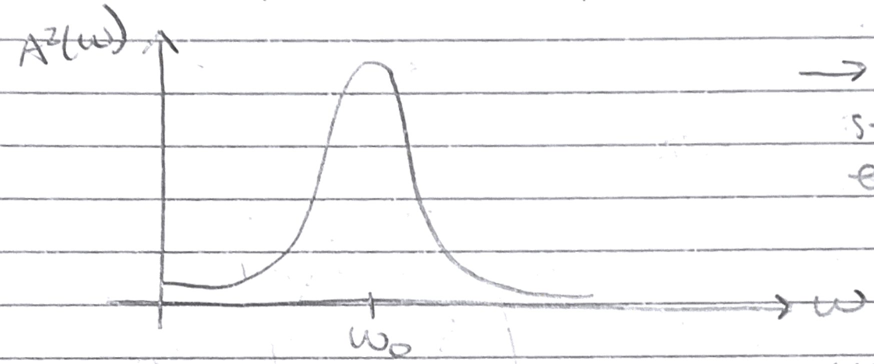
solution w/cosine (driver)

$$\tan(\phi) = \frac{2\beta\omega}{\omega^2-\omega_0^2}$$

*angle of phasor = ωt
ϕ = difference in angles/ωt's



$$A(\omega) = \frac{\mathcal{E}_0/L}{\sqrt{(\omega^2-\omega_0^2)+4\beta^2\omega^2}}$$



→ will respond/store most energy @ driving frequencey = resonance

<u>quality factor</u>: $Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{2\beta} = \frac{1}{LC}\frac{1}{R} = \boxed{\frac{\sqrt{L}}{\sqrt{C}}\frac{1}{R}}$

Review of physics 195: also in chapter 8 of the textbook