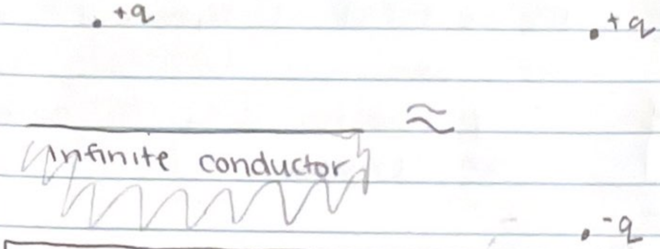
lecture 12: method of images: 2/13/23



this can be done because of boundry conditions

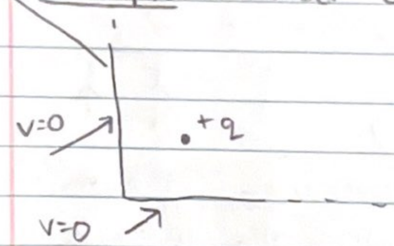
electric fields near conductors

→ electrostatics: charges have come to rest

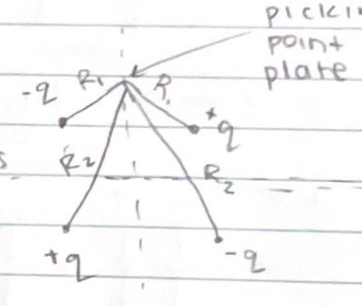
- $\vec{E}=0$ in material of conductor
- excess q on conductor sits on surface

example: method of images

non-uniform negative charge



equiv situation ≈ w/point charges



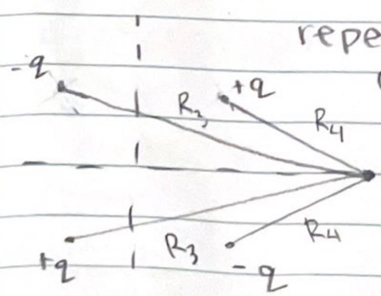
picking arbitrary point where plate would be

mathematical test:

1) $V = \cancel{\frac{k(-q)}{R_1}} + \frac{k(+q)}{R_1} + \frac{k(-q)}{R_2} = \frac{k(-q)}{R_2}$ ☒ DOES NOT WORK, add positive point

2) $V = \cancel{\frac{k(-q)}{R_1}} + \cancel{\frac{k(+q)}{R_1}} + \cancel{\frac{k(-q)}{R_2}} + \frac{k(q)}{R_2} = 0$ ☑ yay!

repeating, using different point for calculating voltage at

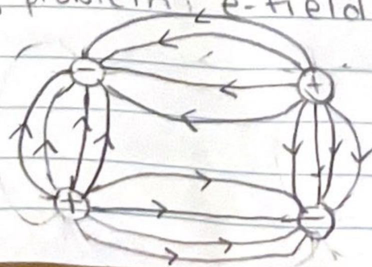


mathematical test:

$V = \cancel{\frac{k(q)}{R_4}} + \frac{k(-q)}{R_4} + \frac{k(q)}{R_3} + \cancel{\frac{k(-q)}{R_3}} = 0$

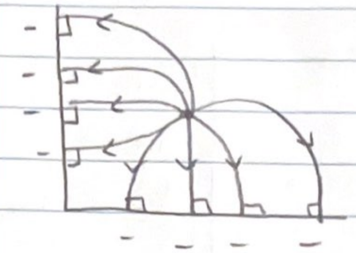
⟹ math once again works!

problem: e-field must be unique



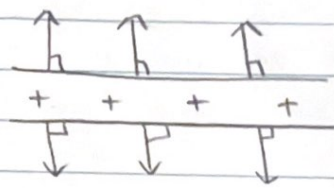
→ ★ no e-field in center of charges

(continued example on back)

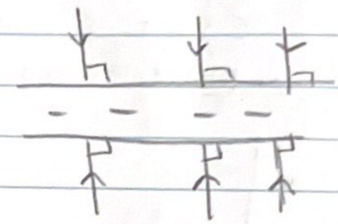


the e-field is symmetrical, and we can use boundry conditions a proportionality to determine the charge.

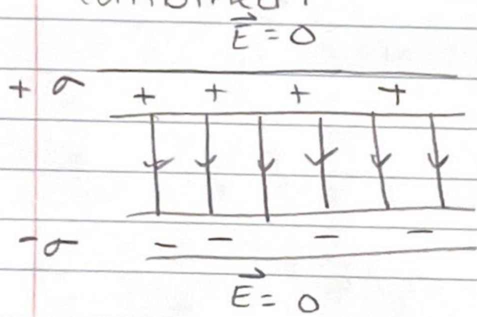
infinite positive plane: (e-fields)



inifinite negitive plane:

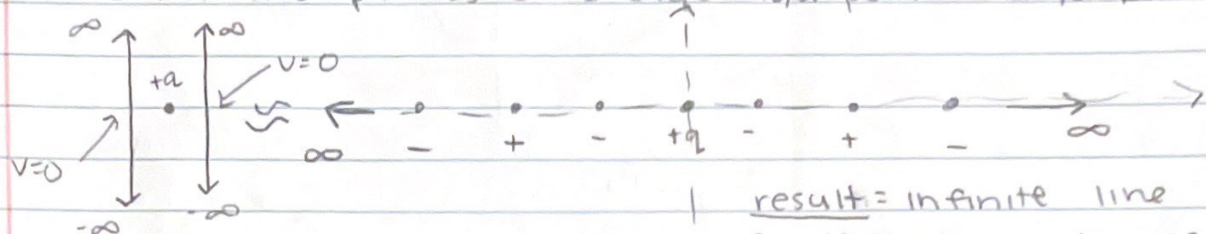


combined:



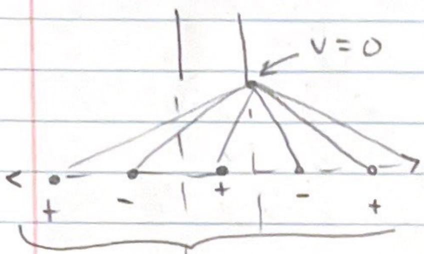
combine together using the principles of super-position method of images example: 2/15/23

→ 2 infinite planes of charge w/a point charge btwn



result = infinite line of alternating charges from either side of q

Check:



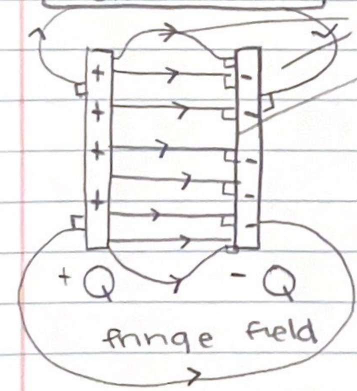
$$V = \frac{k(+q)}{R_1} + \frac{k(-q)}{R_1} + \frac{k(q)}{R_2} + \frac{k(-q)}{R_2} + \ldots$$

⇒ now we can find e-field and voltage everywhere in this space

all the charges pair up and cancel along places where the plates would be

approx. uniform σ (surface charge) on inner surface

real world



two metal plates → charges will be drawn to surface closest to each other generating an e-field

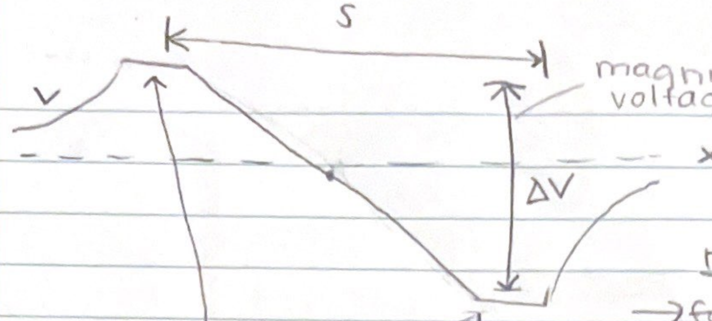
small amount of charge will be repelled from same charge and go to outside of plate, creating a fringe field w/ the outer charge from the other plate's outer charge

fringe field:

anything that does not go straight across. It is very weak and usually ignored. (can be seen on pg. 132 / 143 in textbook)

*example/graph of potential cont. on back

2/15/23



magnitude of voltage across

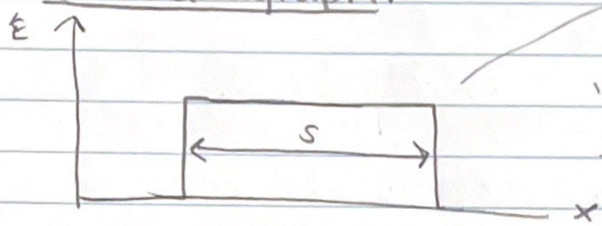
where it hits the capacitor

recall: $\vec{E} = -\vec{\nabla} V$

→ for a uniform $\vec{E}$

$$E_x = \frac{-\Delta V}{\Delta x}$$

<u>e-field graph:</u>



→ derivative of voltage graph

in terms of s:

$$E_x = \frac{|\Delta V|}{s}$$

⇒ capacitors <u>store energy</u>

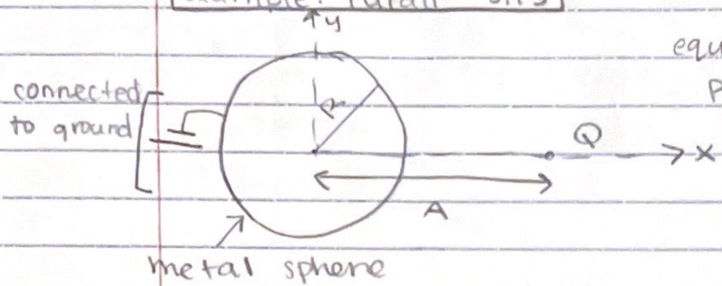
<u>youtube video on capacitors:</u>

what's a capacitor on khan academy

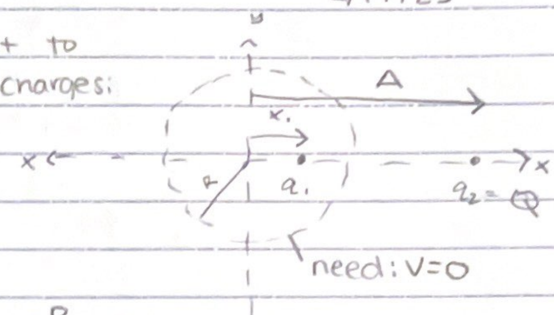
$$C = \frac{Q}{V} = \frac{\text{charge}}{\text{voltage}}$$

*units = Farads = $\frac{\text{coulomb}}{\text{volt}}$

example: Purcell 3.13 2/17/23



equivlant to point charges: $\approx$



claim: if $x_1 = \frac{R^2}{A}$ and $q_1 = \frac{-R}{A}Q$, then it is a method of images.

check: boundry conditions: $\infty \approx \infty$

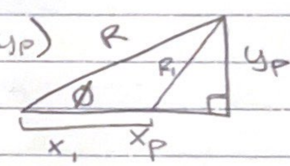
$q_2 = Q \approx q_2 = Q$

$\rightarrow$ testing that there is a sphere of no charge still when using point charges:

- arbitrary point "P" represents any point on sphere

$\rightarrow$ voltage at P =

$$V_p = \frac{kq_1}{R_1} + \frac{kq_2}{R_2} \qquad p:(x_p, y_p)$$



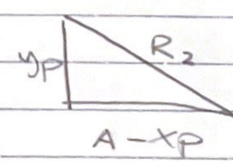
for R_1:

$$R_1^2 = (x_p - x_1)^2 + y_p^2$$

$$\sqrt{R_1^2 - y_p^2} + x_1 = x_p \qquad \sin\phi = \frac{y_p}{R}, \quad \cos\phi = \frac{x_p}{R} \Rightarrow x_p = R\cos\phi, \quad y_p = R\sin\phi$$

$$R_1^2 = (R\cos\phi - x_1)^2 + (R\sin\phi)^2$$
$$= R^2\cos^2\phi - 2x_1 R\cos\phi + x_1^2 + R^2\sin^2\theta$$
$$= R_1^2 + x_1^2 - 2x_1 R\cos\phi$$

for R_2:



$$R_2^2 = (A - x_p)^2 + y_p^2$$
$$= (A - R\cos\phi)^2 + (R\sin\phi)^2$$
$$= A^2 - 2AR\cos\phi + R^2\cos^2\phi + R^2\sin^2\phi$$
$$= A^2 - 2AR\cos\phi + R^2$$

plugging $\rightarrow$ back into equation for voltage:

$$V_p = \frac{kq_1}{\sqrt{R^2 + x_1^2 - 2xR\cos\phi}} + \frac{kq_2}{\sqrt{A^2 - 2AR\cos\phi + R^2}}$$

$$= \frac{k\left(-\frac{R}{A}Q\right)}{\sqrt{R^2 + \left(\frac{R^2}{A}\right)^2 - 2\frac{R^2}{A}\cos\phi}} + \frac{kQ}{\sqrt{A^2 - 2AR\cos\phi + R^2}}$$

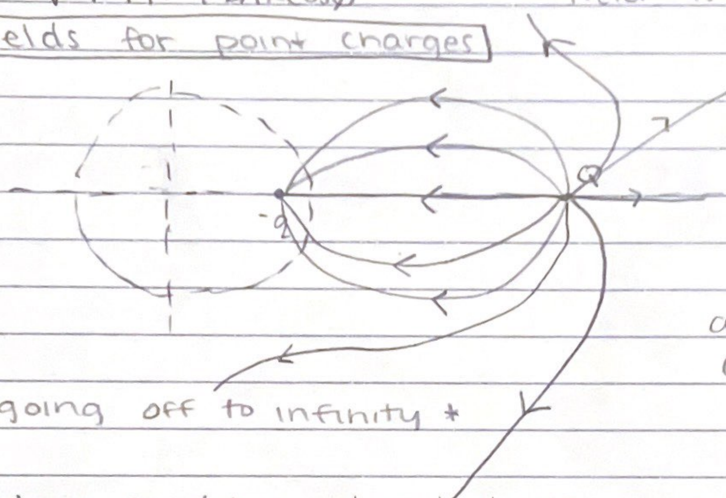
(continued on the back)

$$V_P = \frac{k\left(\frac{-R}{A}\right)Q}{\sqrt{\left(\frac{R^2}{A^2}\right)\left[A^2+R^2-2AR\cos\phi\right]}} = \cancel{\left(\frac{A}{R}\right)} \frac{k\left(\cancel{\frac{-R}{A}}\right)Q + kQ}{\sqrt{A^2+R^2-2AR\cos\phi}}$$

$$= \frac{KQ - KQ}{\sqrt{A^2+R^2+2AR\cos\phi}} = 0$$

yay! proof that the field is zero on disk

$\boxed{\text{e-fields for point charges}}$



$q = \frac{-R}{A} Q$.

⇒ whole thing is net positive, will ~~have to~~ look like a single point charge from far away

going off to infinity

⇒ khan academy: dialectrics in capacitors (find in physics: circuits section of khan academy)