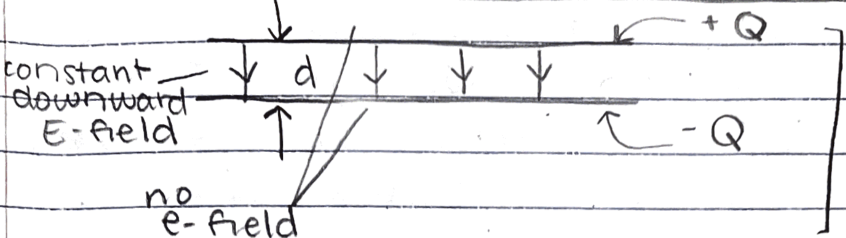
Capacitance: $C = \frac{Q}{V}$ w/units $\underbrace{1F = 1\frac{C}{V} = \frac{1C^2s^2}{kgm^2}}_{\text{quite large}}$ 2/20/23

(typically look at microfarads)

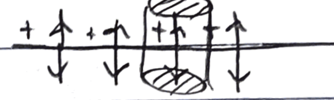
GEOMETRY MATTERS!!

ex: (ideal) parallel plate capacitor:



assume the plates have an area "A"

(1 plate):

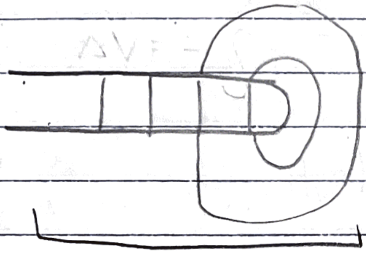


magnitude of e-field btwn plates:

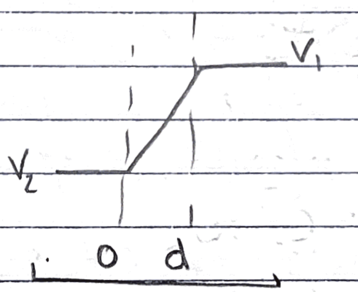
(1 plate): $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} \Rightarrow 2EA$

$E = \frac{\sigma}{2\epsilon_0}$

Therefore for 2-plates: $E = \frac{\cancel{2}\sigma}{\cancel{2}\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$



e-field lines of capacitor



graph of potential

$\Delta V = -\int \vec{E} \cdot d\vec{a}$

$= -E(-d)$

$= \boxed{\frac{\sigma}{\epsilon_0} d}$

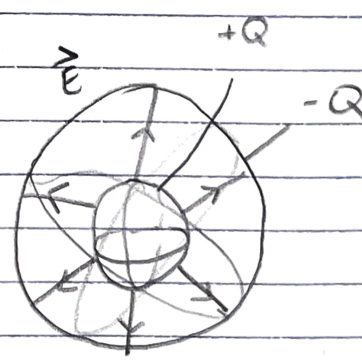
$C = \frac{Q}{\Delta V} = \frac{\epsilon_0 Q}{\sigma d} = \frac{\epsilon_0 \cancel{Q} A}{\cancel{Q} d} = \boxed{\frac{\epsilon_0 A}{d}}$ solution for capacitance of a parallel plane capacitor

★ C always has E

★ It depends on geometry/shape

★ has length scale (ex: $\frac{A}{d}$)

example 2: sphereical shell cap:



$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

$\Delta V = -\int \vec{E} \cdot d\vec{l} = -\int_b^a \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{Q}{r^2}\right) dr$

$= -\left(\frac{Q}{4\pi\epsilon_0}\right)\left[-\frac{1}{r}\right]_a^b = \left(\frac{Q}{4\pi\epsilon_0}\right)\left(-\frac{1}{a} + \frac{1}{b}\right)$

$= \left(\frac{Q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)\right)$

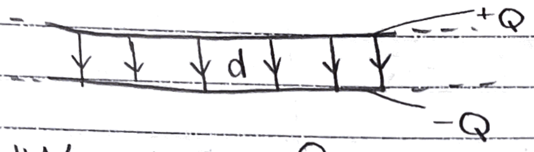
capacitance from book

$C = \frac{Q}{\Delta V} = \left(\frac{4\pi\epsilon_0 Q}{Q}\right)\left(\frac{ab}{b-a}\right) = \boxed{\frac{4\pi\epsilon_0 ab}{a-b}}$

* DNE can compute "C" for arbitrary configurations of conductors *

→ C is found in a pairwise, C_{12}, fashion

<u>energy in (parallel plate) capacitors:</u>



lets take dQ charge from the lower plate and lift it to the top plate

the work: $dW = V\,dQ = \frac{Q}{C}\,dQ$

Potential energy

for total charge Q: $W = \int dW = \int_0^Q \frac{1}{C} Q\,dQ = \left(\frac{1}{C}\right)\int_0^Q Q\,dQ = \boxed{\left(\frac{1}{C}\right)\left(\frac{Q^2}{2}\right) = U}$

different forms:

$$U = \left(\frac{1}{C}\right)\left(\frac{Q^2}{2}\right) = \frac{1}{C}\left(\frac{C^2V^2}{2}\right) = \boxed{\frac{1}{2}CV^2}$$

for parallel plate capacitors:

$$U = \left(\frac{1}{2}\right)\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \left(\frac{1}{2}\right)(\epsilon_0)\underbrace{(Ad)}_{\text{volume}}(E^2)$$

⇒ the energy density of the electric field is $\boxed{\frac{1}{2}\epsilon_0 E^2}$

$\vec{F} = -\nabla U$ → for 1D: $F_x = \frac{dU}{dx}$

apply a force to change the capacitor

$F_{applied} = \frac{d}{dx}U = \frac{Q^2}{2}\frac{d}{dx}\left(\frac{1}{C}\right)$

⇒ to change the shape of C

★MID-TERM:
Thursday evening
7pm
→ review soon!

2/22/23

last time: capacitors:

* what is the capacitance?

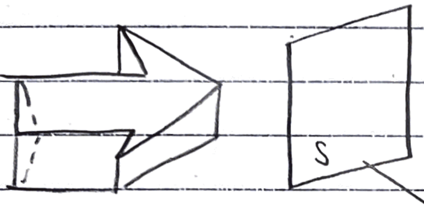
→ for parallel plate capacitors ||:

$$C = \frac{\epsilon_0 A}{d} \longrightarrow \underbrace{\frac{\epsilon A}{d} = \frac{\epsilon_0 (1+\chi) A}{d}}_{\text{with dielectric material}}$$

→ in circuits: $Q = CV$

→ energy stored in capacitors: $U = \frac{1}{2}\frac{Q^2}{C}$

Today: Tour of circuits



$$\boxed{I = \int_S \vec{J} \cdot d\vec{a}}$$

current

continuity equation

$$\boxed{\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

charge density

$\vec{J}$ = current density

surface

- generally: $\boxed{J = \sigma E}$ (locally)

! NOT CHARGE = CONDUCTIVITY !

$$\boxed{I = \frac{V}{R}}$$

(across an entire wire; globally)

OHM'S LAWS

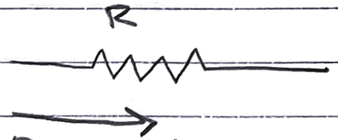
units:

$V = IR$

$R = \text{Ohm} = \Omega = \frac{1V}{1\text{amp}} = \frac{1\,kg\,m^2}{Cs}$

-any bit of conductor that is not a superconductor has some sort of resistance.

→ charge going through a resistor:



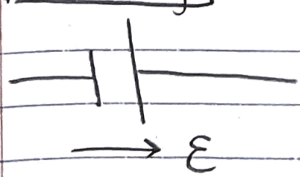
for a little charge flow in dt:

$U = (I\,dt)V = (I\,dt)IR$

$$\boxed{\text{power} = P = \frac{dU}{dt} = I^2 R}$$

$dQ = I\,dt$

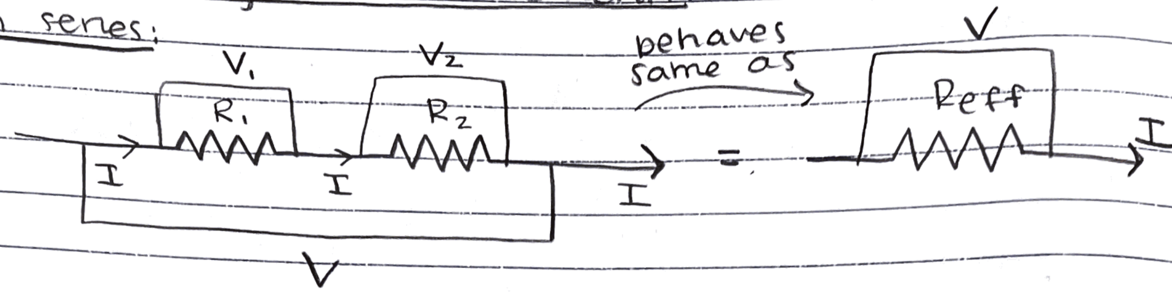
Battery drawn like this!



(will come back to batteries in more depth later in semester!)

combining circuit elements:

in series:

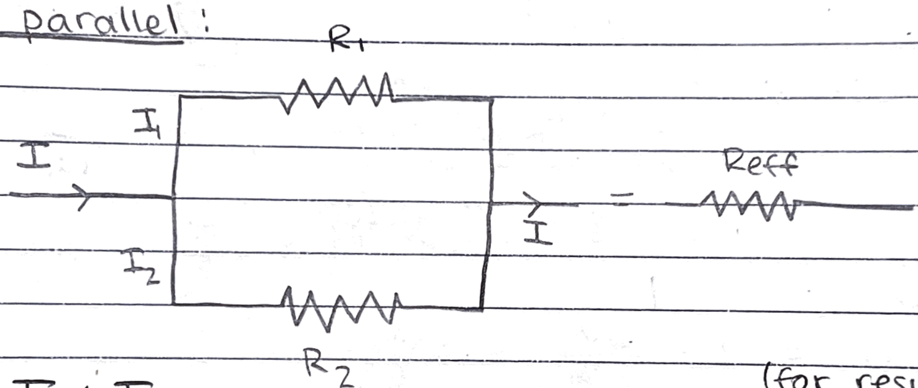


$$V = IR_{eff} = IR_1 + IR_2$$
$$V = V_1 + V_2$$

currents are constant; $\boxed{R_{eff} = R_1 + R_2}$ (for resistors in series)

★in series, resistance is added directly★

in parallel:



$I = I_1 + I_2$

$V = V_1 = V_2$ $\Rightarrow$ $\dfrac{V}{R_{eff}} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$ $\Rightarrow$ $\boxed{\dfrac{1}{R_{eff}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}}$ (for resistors in parallel)

$V = IR$

★in parallel, resistance is added inversley★

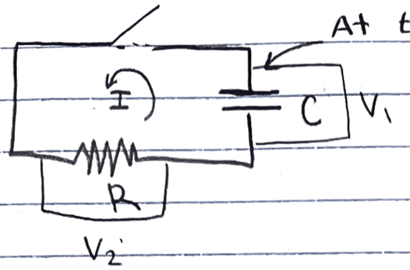
KIRCHHOFF'S RULES

1) resistors behave as $V = IR$
2) for every node, currents are conserved
3) for every loop, voltages sum to zero $\left(\oint \vec{E} \cdot d\vec{s} = 0\right)$

Circuit example problems

resistor + capacitor (one loop)

(1)



At $t=0$, charged at Q_0

$V_1 = \frac{Q}{C}$

$\frac{Q}{C} + IR = 0$

$V_2 = IR$

but, $I = \frac{dQ}{dt}$, which implies:

$\frac{dQ}{dt} + \frac{Q}{RC} = 0$

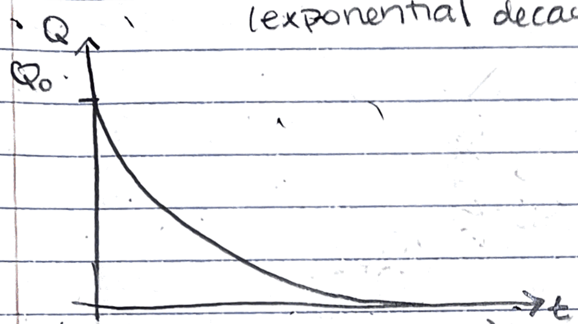
$\frac{dQ}{dt} = \frac{-Q}{RC} \Rightarrow \frac{dQ}{Q} = \frac{-dt}{RC}$

$= \int_{Q_0}^{Q(t)} \frac{dQ}{Q} = -\int_{Q_0}^{Q(t)} \frac{dt}{RC}$

$= \ln(Q)\Big|_{Q_0}^{Q(t)} = \frac{-t}{RC}$

graphically:

(exponential decay)



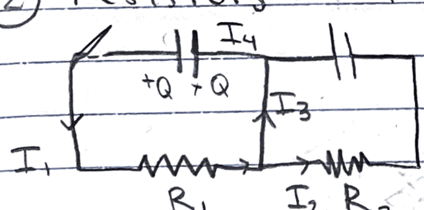
$\ln(Q)\Big|_{Q_0}^{Q(t)} = \ln(Q(t)) - \ln(Q_0)$

$= \ln\left(\frac{Q(t)}{Q_0}\right)$

$\Rightarrow$ exponentiate: $\frac{Q(t)}{Q_0} = e^{-\frac{t}{RC}}$

$\boxed{Q(t) = Q_0 e^{-t/RC}}$

(2) resistors + capacitors (two loops)



$I_1 = I_2 + I_3$

$I_1 = I_4$

loops

$\frac{Q_1}{C} + IR = 0$

$\frac{Q_2}{C} + IR_2 = 0$

$I_3 = I_1 - I_2$

$I = \frac{dQ}{dt}$

$V_1 + V_2 = 0$

$V_3 + V_4 = 0$

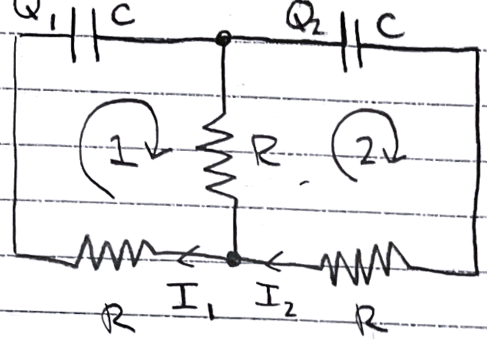
$\frac{Q_1}{C} + IR_1 = \frac{Q_2}{C} + IR_2 = 0$

*signs for R: potential drops across resistors

$-IR$ if in direction of I

$+IR$ if in opposite direction of I

example continued w/a resistor:



nodes: $I_1 = I_2 + I_3 \Rightarrow I_3 = I_1 - I_2$

loops: $\begin{cases} -I_1R + \frac{Q_1}{C} - I_3R = 0 \\ -I_2R + I_3R + \frac{Q_2}{C} = 0 \end{cases}$

$\Rightarrow \begin{cases} -I_1R + \frac{Q_1}{C} - (I_1 - I_2)R = 0 \\ -I_2R + (I_1 - I_2)R + \frac{Q_2}{C} = 0 \end{cases}$

$I = I_1 + I_2$

$Q = Q_1 + Q_2$

adding +

$-(I_1 + I_2)R + \frac{Q_1 + Q_2}{C} = 0$

$\boxed{IR + \frac{Q}{C} = 0}$

expect a solution like last:

$\boxed{Q(t) = Q_{sum_0} e^{-t/RC}}$

$= Q_1(t) + Q_2(t)$

subtracting:

$= (-I_1 + I_2)R + \frac{Q_1 - Q_2}{C} - 2(I_1 - I_2)R = 0$

$= \boxed{3(I_1 - I_2)R + \frac{Q_1 - Q_2}{C} = 0}$

$\boxed{3IR + \frac{Q}{C} = 0}$

$\boxed{Q_s = Q_{diff_0} e^{-t/3RC}}$

similar expected solution as for adding

This means:

$Q_1(t) = ae^{-t/RC} + be^{-t/3RC}$

$Q_2(t) = ae^{-t/RC} - be^{-t/3RC}$

$\boxed{\text{at } t=0}$

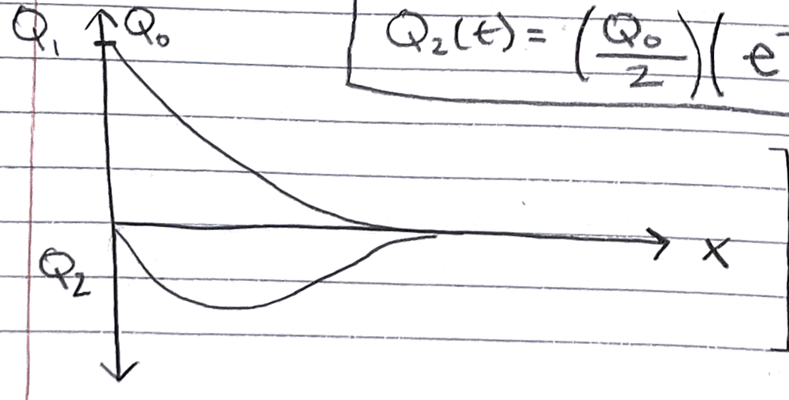
$Q_1(t) = Q_0 = a + b$

$Q_2(0) = 0 = a - b$

Therefore:

final solutions!

$$\boxed{\begin{aligned} Q_1(t) &= \left(\frac{Q_0}{2}\right)\left(e^{-t/RC} + e^{-t/3RC}\right) \\ Q_2(t) &= \left(\frac{Q_0}{2}\right)\left(e^{-t/RC} - e^{-t/3RC}\right) \end{aligned}}$$



graphically looks like this

special relativity:

based on:

→ the speed of light is independant on the motion of source and observer

$\boxed{= \text{consistancy of } c}$

→ "Nature phenomena run their course according to exactly the same general laws... in all inertial reference frame"

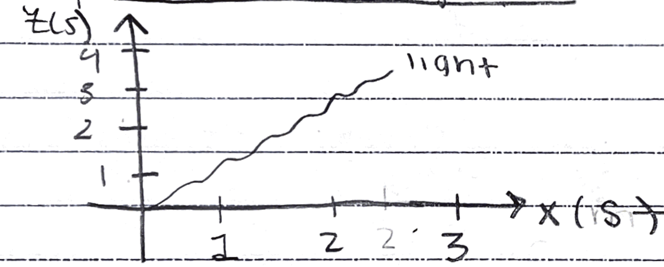
$\boxed{= \text{relativity principle}}$

results:

- time dialation
- length contraction
- loss of simultaneity

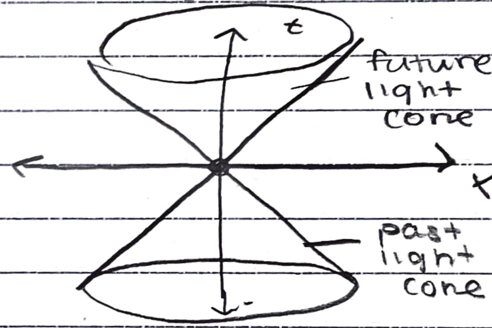
$\boxed{\text{what is simultaneous?}}$

space-time diagrams:



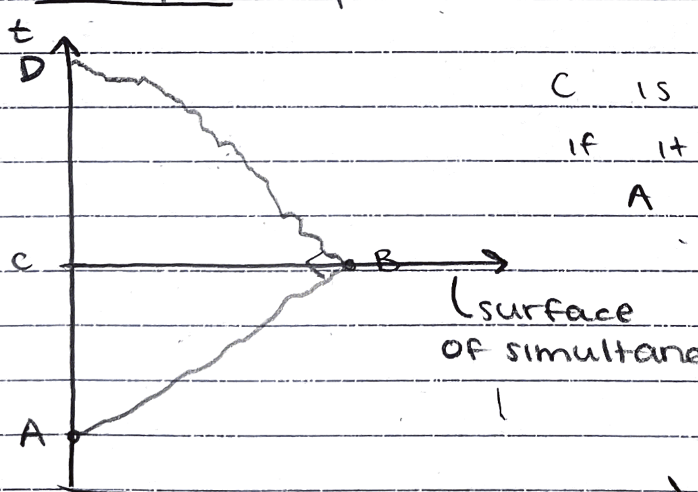
light-seconds! yay!

*inertial reference frame = non-accelerating



light travels on diagonal lines in s-t diagrams

example: I part I



surface of simultaneaty

light being sent + reflected back

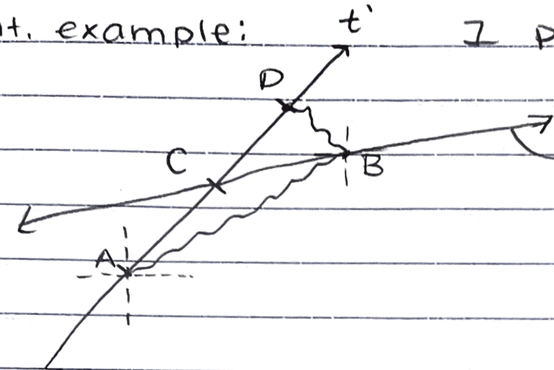
C is simultaneous with B if it is halfway between A and D:

$$T_C = \frac{T'' - T}{2} + 2 = \frac{T'' + T}{2} = T_B$$

by definition

relys on c being constant

cont. example: 1 part 2

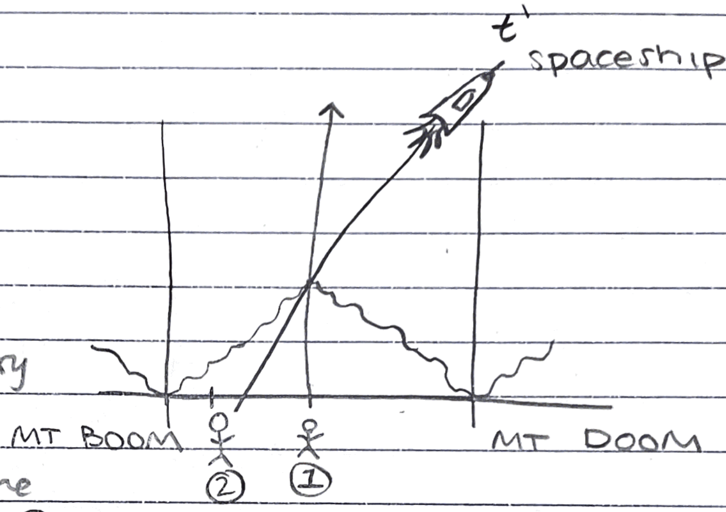


C + B = simultaneous

surface of simultaneity

puzzle:

two volcaneos erupt at same time on either side of stationary observer



⇒ eruptions are simultaneous 1

⇒ BOOM = first then DOOM, seeing light, still times = simultaneous 2