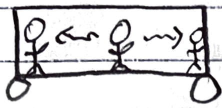
last time: 2/27/23

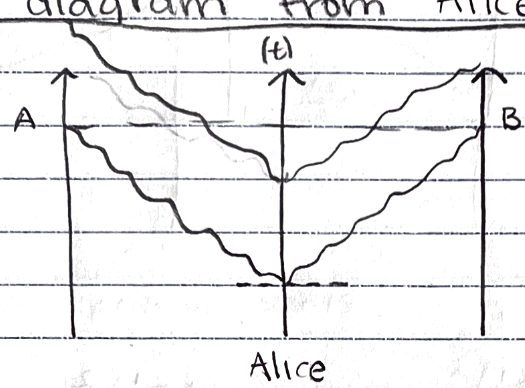
circuit example, special relativity → definition of simultaneity

- slip in simultaneity



[alice in center of train and wants simultaneous sparks at each end of train: need way of timing

S-T diagram from Alice's POV:

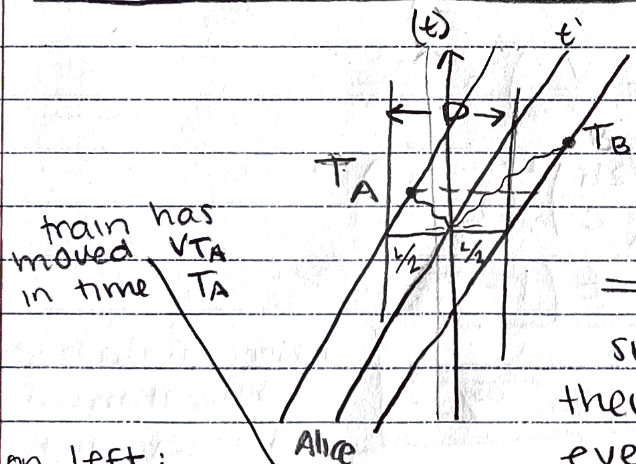


send signal w/light

→ events = simultaneous

events are simultaneous

another inertial reference frame: (passing Alice outside train)



event A now appears to occur before event B

⇒ if two events are simultaneous in one frame, they are not simultaneous in every other frame ⇐

on left:

$CT_a = \frac{L}{2} - VT_A$

on right:

$CT_B = \frac{L}{2} + VT_B$

And $D = c(T_A + T_B)$

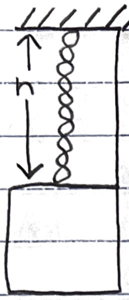
"slip in simultaheity"

what is $\Delta T = T_B - T_A$?

$c(T_B - T_A) = \cancel{\frac{L}{2}} - \cancel{\frac{L}{2}} + (V(T_B + T_A))$ combining above equations

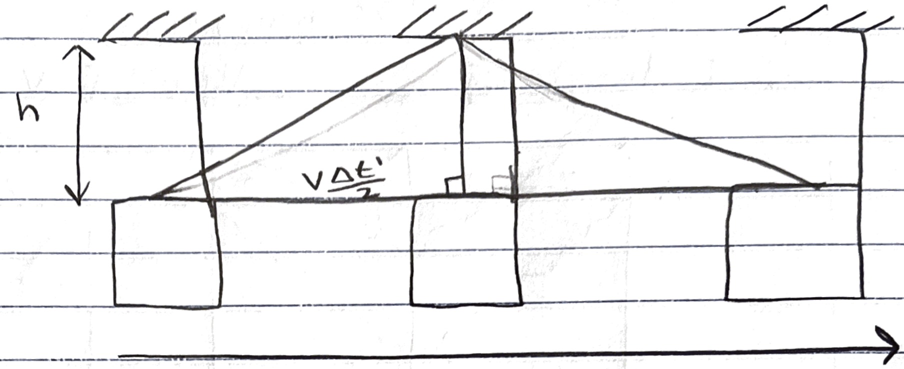
$c\Delta T = \frac{VD}{c} \Rightarrow \boxed{\Delta T = \frac{VD}{c^2}}$

example 3: light Clock



one tick: $\Delta t = \frac{2h}{c}$
(in proper frame)

<u>in a new frame, moving right:</u>



clock moving right ⊥ to light path

using pythagorean's theorem:

$$\left(\frac{c\Delta t'}{2}\right)^2 = h^2 + \left(\frac{v\Delta t'}{2}\right)^2$$

$$= \left(\frac{\Delta t'}{2}\right)^2\left(1-\frac{v^2}{c^2}\right) = \frac{h^2}{c^2}$$

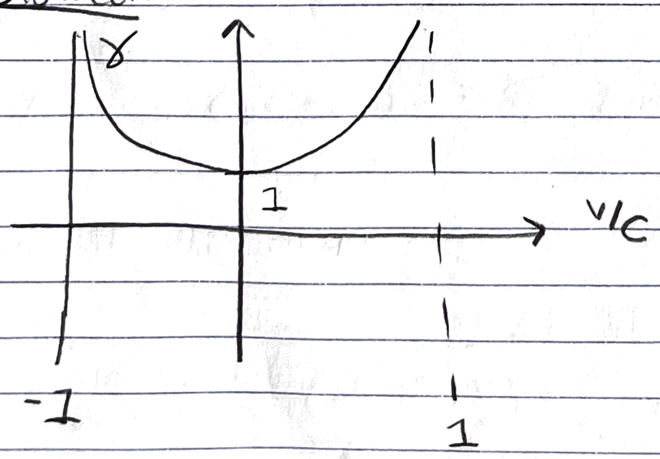
$$(\Delta t')^2 = \left(\frac{2h}{c}\right)^2\left(1-\frac{v^2}{c^2}\right)^{-1}$$

$$\Delta t' = \Delta t\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$\boxed{\gamma = \frac{1}{\sqrt{1-v^2/c^2}}}$$

time dialation
→ light must travel longer distance for one tick

<u>plotted:</u>



"moving clocks run slow"

$\boxed{\Delta t' = \gamma \Delta t}$

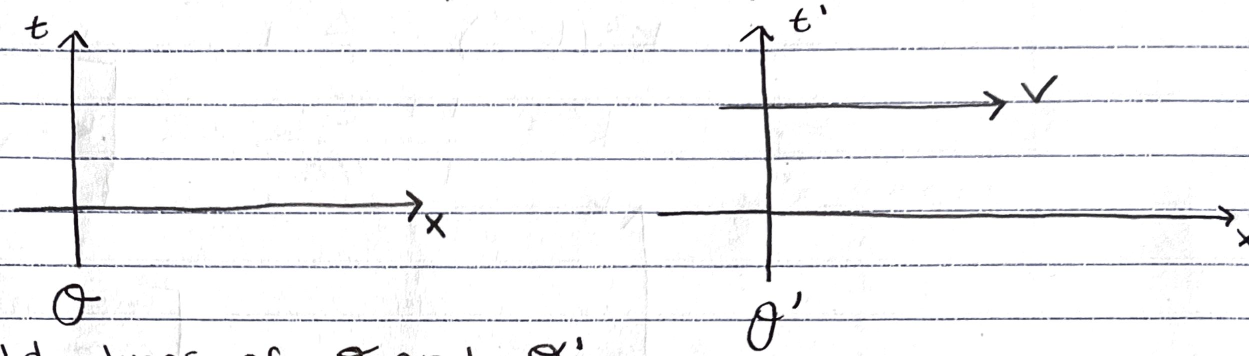
! COMMIT GAUSS' LAW TO MEMORY → MUST KNOW ALL MAXWELL EQ !

last time: special relativity - slip in simultaneity 3/1/23

$\Delta t = \frac{VD}{c^2}$ time dilation: $t' = \gamma t$

★ special relativity reading: Ellis and williams "Flat and curved spacetimes"

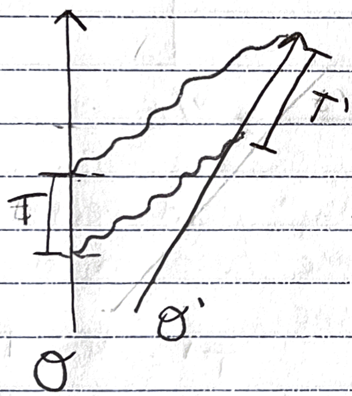
RED-SHIFT: consider two frames:



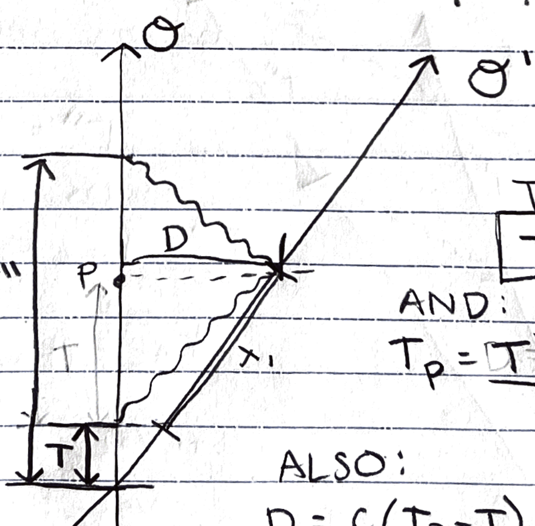
world lines of O and O'

→ define K as ratio of T' to T

$T' = KT$



How is K related to V? (assuming $V > 0$ = recession) drawn larger:



$T' = KT$

$T'' = KT'$

$\boxed{T'' = K^2 T}$

AND:

$T_P = \frac{T'' - T}{2} + T = \boxed{\frac{T'' + T}{2}}$

ALSO:

$D = c(T_P - T) = \boxed{\left(\frac{T'' - T}{2}\right) c}$

AND: $D = VT_P = \boxed{V\left(\frac{T'' + T}{2}\right)}$

(math continued on back)

plugging in: 3/1/23

$$\frac{c(k^2-1)\cancel{T}}{\cancel{2}} = \frac{v(k^2+1)\cancel{T}}{\cancel{2}}$$

$$c(k^2-1) = v(k^2+1)$$

$$k^2 - 1 = \frac{v}{c}(k^2) + \frac{v}{c}$$

$$k^2\left(1-\frac{v}{c}\right) = \frac{v}{c} + 1$$

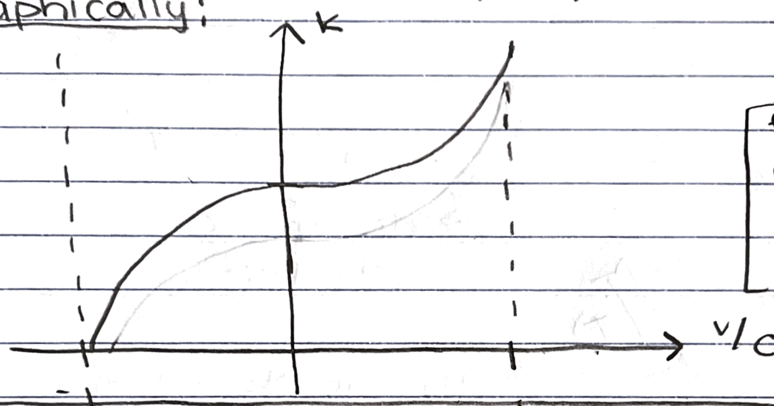
$$k^2 = \frac{1+v/c}{1-v/c} \quad \text{OR}$$

DOPPLER EFFECT

$$\boxed{k = \sqrt{\frac{1+v/c}{1-v/c}}}$$

RED SHIFT FACTOR

graphically:

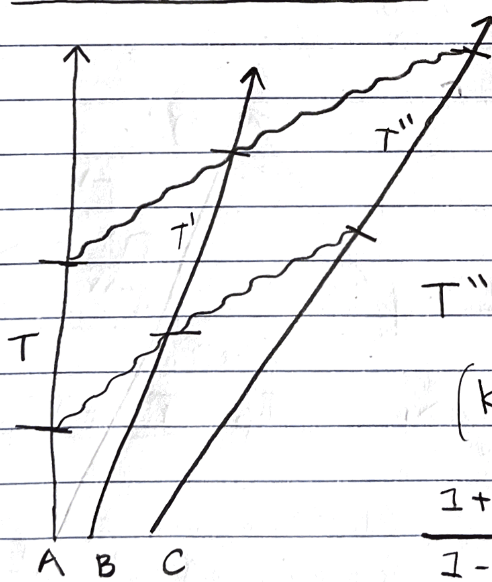


[for opposite: approaching observers →

$$k_{app} = \sqrt{\frac{1-v/c}{1+v/c}} = \frac{1}{k_{recession}}$$]

velocity addition: "normally" add vectors: $\vec{V}_{AC} = \vec{V}_{AB} + \vec{V}_{BC}$

NOT the case in special relativity



by def:

$$T' = k_{AB}T$$ velocity btwn A & B

$$T'' = k_{AC}T = k_{BC}T'$$

which implies:

$$T'' = k_{BC}k_{AB}T$$

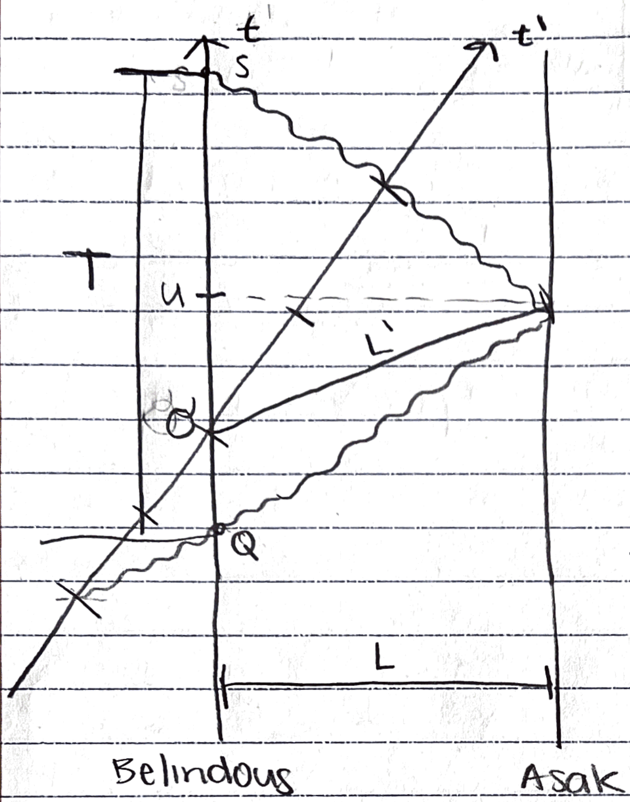
$$(k_{AC})^2 = (k_{AB}k_{BC})^2$$

$$\frac{1+\frac{V_{AC}}{c}}{1-\frac{V_{AC}}{c}} = \left(\frac{1+\frac{V_{AB}}{c}}{1-\frac{V_{AB}}{c}}\right)\left(\frac{1+\frac{V_{BC}}{c}}{1-\frac{V_{BC}}{c}}\right)$$

$$\rightarrow \boxed{V_{AC} = \frac{V_{AB}+V_{BC}}{1+\frac{V_{AB}V_{BC}}{c^2}} \leq c}$$

Lorentz contractions

- length contraction:



$T = t_{QS} = t_{QO} + t_{OS}$

$t_{QO} = \frac{1}{k}\frac{T'}{2}$ relations

$t_{OS} = k\frac{T'}{2}$

$k = \sqrt{\frac{1+v/c}{1-v/c}}$ *for recession

$k = 1/k$ for approach

$T = \left(\frac{1}{k} + k\right)\frac{T'}{2}$

$= \left(\frac{k^2+1}{2k}\right)T'$

$L' = \frac{c(T')}{2}$ AND $L = \frac{cT}{2}$

therefore:

$$\frac{L'}{L} = \frac{T'}{T} = \frac{2k}{k^2+1} = \frac{2\sqrt{\frac{1+v/c}{1-v/c}}}{\frac{1+v/c+1-v/c}{1-v/c}} = \frac{\cancel{2}\sqrt{\frac{1+v/c}{1-v/c}}}{\left(\cancel{2}/_{1-v/c}\right)}$$

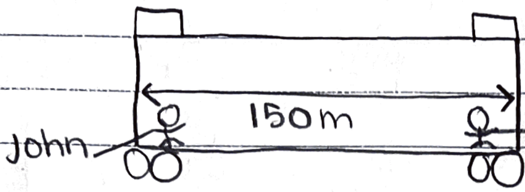
$$= \sqrt{\frac{1+v/c}{1-v/c}}\,(1-v/c)^{1/2} = \sqrt{(1+v/c)(1-v/c)}$$

$$= \sqrt{1-(v/c)^2} = \boxed{\frac{1}{\gamma}}$$

so:

$$\boxed{L' = \frac{1}{\gamma}L}$$ "moving objects shrink"

example 2: very long train-car w/skylights 3/3/23



- have synchronized watches at either end of the train

→ train goes through 150m tunnel at $\frac{3}{5}c$

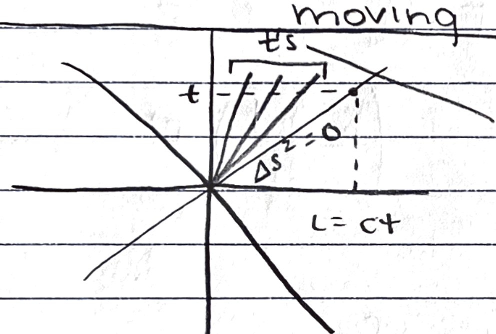
$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-9/25}} = \frac{1}{\sqrt{\frac{16}{25}}} = \frac{5}{4}$$

(relativistic)

Length tunnel $= \frac{150}{\gamma} = \frac{4}{5}(150) = 120m$

$L' = \frac{L}{\gamma}$

→ according to the train the tunnel is moving and therefore contracted



Remember:
"moving clocks run slow"

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ as $v \to c$, $\gamma = 1$

everything happens simultaneously for photons

$$\boxed{\Delta s^2 = -c^2\Delta t^2 + \Delta x^2}$$

seperation in space-time

example cont: moira is watching the train go through a tunnel from a point of rest in respect to the tunnel.

$L_{train} = \frac{150}{\gamma} = 120m$

→ In moira's frame, John & Wendy look up at different times, whereas in John + Wendy's frame they look up simultaneously.