before break: special relativity 3/27/23

→ today: magnetics (chapter 5)

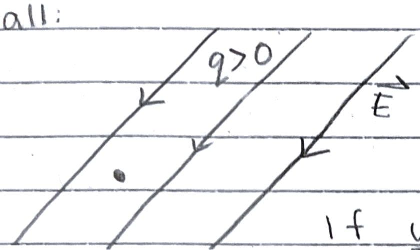
looking ahead: guide 7 (due Friday)

Office hours on THR will end @ 4pm

*Next Friday: Brian will teach on building E & B fields for experiments.

→ for fun: building motors

Recall:

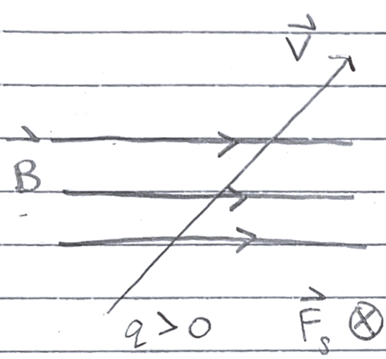


$\vec{F}_E = q\vec{E}$ (how we've defind $\vec{E}$)

→ lots of experiments led us to:

If we have a $\vec{B}$-field, then:

$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$ (magnetic force)



*remember right-hand-rule: hand in direction of vector, fingers curled in direction of magnetic field, thumb in direction of magnetic force *

units: $\left(\frac{\vec{F}_B}{qv}\right) = (B) = \frac{Ns}{Cm}$ = SI unit of magnetic field = tesla

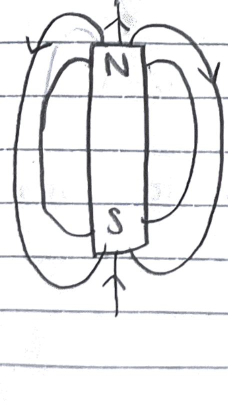
$= \frac{kgms}{s^2Cm} = 1T$ (one tesla = very large)

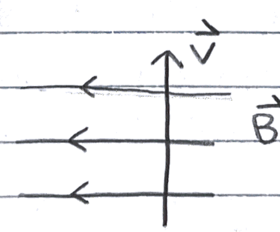
$1G = 10^{-3}T$

Guass' field

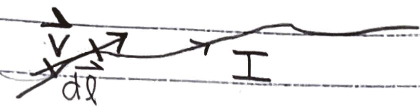
demo:

① magnents:





demo (2): car battery + wires



→ what is the force on a wire? (in uniform B-field)

$$d\vec{F} = dq\,\vec{v}\times\vec{B} = dq\,\frac{d\vec{\ell}}{dt}\times\vec{B}$$

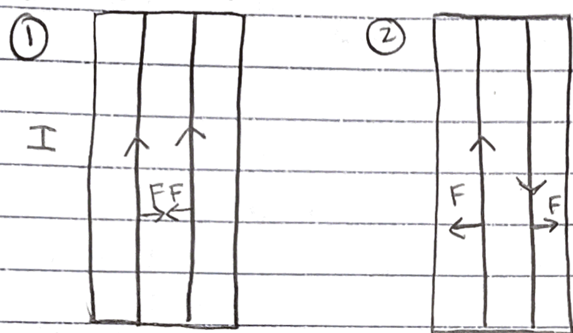
$$= \frac{dq}{dt}\,d\vec{\ell}\times\vec{B} = I\,d\vec{\ell}\times\vec{B}$$

(to create simple form):
suppose $d\vec{\ell}$ is the same direction along the wire

$$\vec{F} = \int d\vec{F} = \int I\,d\vec{\ell}\times\vec{B} = I\int d\vec{\ell}\times\vec{B} = I\vec{\ell}\times\vec{B}$$

two wires connected to car battery:

$$\boxed{F_B = I\vec{\ell}\times\vec{B}}$$



these experiments allowed
⇒ these forms to be derived:

$\vec{F_B} = I\vec{\ell}\times\vec{B}$

$\vec{F_B} = q\vec{v}\times\vec{B}$

$I\vec{\ell}\times\vec{B}$

using RHR:

$\vec{B} = \odot$ on left wire

$\vec{B} = \otimes$ on right wire

For $\vec{E}$ and $\vec{B}$ the total force on q is:

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

suppose we have $\vec{E}$ and $\vec{B}$ fields in a reference frame S, switching frames to S', q is at rest,
→ does $\vec{F}$ change? $F_B \to 0$ in S', what is happening?

if $\vec{F_B} \to 0$, maybe $\vec{F_E} = q\vec{E}$ changes to compensate when switching from S to S'

(continued on next page!)

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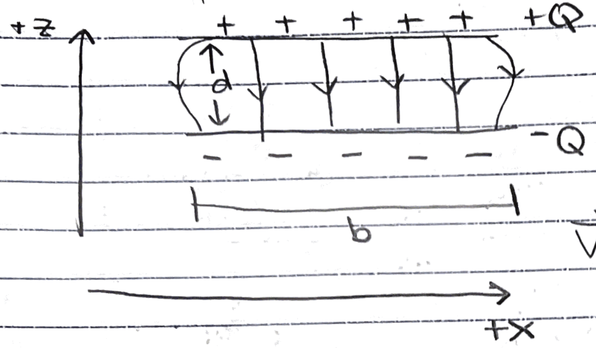
cont:

Idea: charge invariance

"Gauss' Law Holds"

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0} \qquad \oint \vec{E}' \cdot d\vec{a}' = \frac{q_{encl}}{\epsilon_0}$$

example: parallel plate capacitor



$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}}$$

$\vec{V} = -v\hat{i}$ (whole thing moves left)

moving:

parallel plates length contract:

$L' = \frac{L}{\gamma}$ or $b' = \frac{b}{\gamma}$ in x-direction

$A' = bb' = \frac{b^2}{\gamma}$ → what is $\vec{E}'$?

$\boxed{\vec{E}' = \frac{\sigma'}{\epsilon_0}}$ → nothing changes, can still use GAUSS' LAW, there is just a new charge density

$$\sigma = \frac{Q}{A}$$

$\boxed{\vec{E}' = \gamma E}$

$$\sigma' = \frac{Q}{A'} = \frac{\gamma Q}{b^2} = \gamma\sigma$$

Last time:

$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$

special relativity connects these two

Gauss' Law review: $\oint \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}$

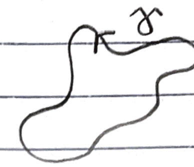
$\boxed{\text{capacitor example}}$: charge invariant: $\oint_S \vec{E} \cdot d\vec{a} = \oint_{S'} \vec{E}' \cdot d\vec{a}'$

→ flux is also invariant

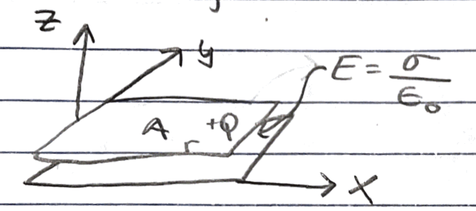
→ another way to find charge = potential

$\Delta V = -\int \vec{E} \cdot d\vec{s}$

$\oint_{\gamma} \vec{E} \cdot d\vec{s} = 0$



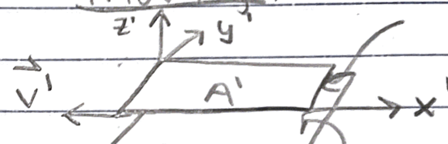
stationary:



moving:

$A' = b'b = \frac{b^2}{\gamma}$

$\sigma' = \frac{Q}{A'} = \frac{\gamma Q}{b^2}$

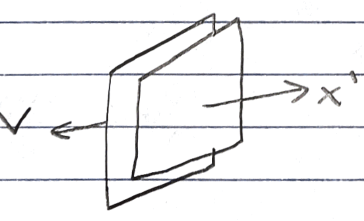


$E' = \frac{\sigma'}{\epsilon_0} = \gamma\sigma$

⇒ therefore $E'_z = \gamma E_z$

or $\boxed{E'_\perp = \gamma E_\perp}$ (perpendicular component to velocity)

in the other direction: flipping capacitor



Now $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{\imath}$

$\vec{E}' = \frac{\sigma'}{\epsilon_0} \hat{\imath}' = \frac{\sigma}{\epsilon_0} \hat{\imath} = \vec{E}$

OR $\boxed{\vec{E}'_{\parallel} = E_{\parallel}}$ (parallel component to velocity)

$|\vec{E}|$ is minimized in the rest frame

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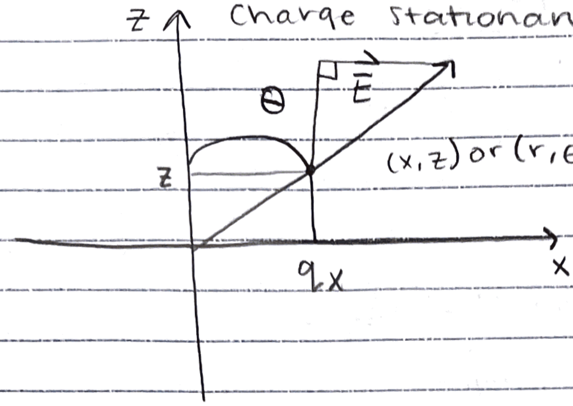
how about for a point charge? — what does field look like?

Charge stationary:



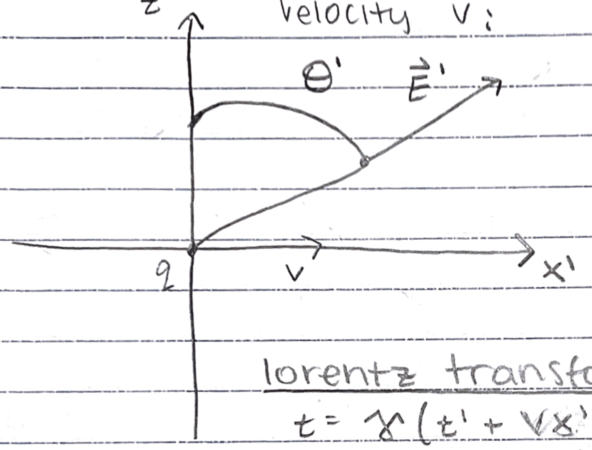
$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}$$

$$E_x = |\vec{E}|\sin\theta = \frac{q}{4\pi\epsilon_0}\frac{1}{(x^2+z^2)}\frac{x}{\sqrt{x^2+z^2}}$$

$$\boxed{= \frac{qx}{4\pi\epsilon_0(x^2+z^2)^{3/2}}}$$

$$E_z = |\vec{E}|\cos\theta = \boxed{\frac{qz}{4\pi\epsilon_0(x^2+z^2)^{3/2}}}$$

charge moving right at velocity v:



$$E_z' = \gamma E_z = \frac{\gamma q z}{4\pi\epsilon_0(x^2+z^2)^{3/2}}$$

$$E_x' = E_x = \frac{qx}{4\pi\epsilon_0(x^2+z^2)^{3/2}}$$

lorentz transformation to extract more:

$t = \gamma(t' + vx')$ $c = 1,\ \beta = v/c$] book notation

$x = \gamma(x' + vt')$

$y = y'$

$z = z'$

→ we want to find $\vec{E}'$ when $t' = 0$:

$$E_z = \frac{\gamma q z}{4\pi\epsilon_0(x^2+z^2)^{3/2}} = \frac{\gamma q z'}{4\pi\epsilon_0(\gamma^2 x'^2 + z'^2)^{3/2}}$$

$$E_x = \frac{qx}{4\pi\epsilon_0(x^2+z^2)^{3/2}} = \frac{\gamma q x'}{4\pi\epsilon_0(\gamma^2 x'^2 + z'^2)^{3/2}}$$

★magnitude of $\vec{E}' = (E_z')^2 + (E_x')^2$

let's compute:

$$\frac{|E'|^2}{(q/4\pi\epsilon_0)}$$ (on next page)

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$$\frac{|E'|^2}{(q/4\pi\epsilon_0)^2} = \frac{\left(\frac{1}{1-v^2}\right)\left(z'^2 + x'^2\right)}{\left(\left(\frac{1}{1+v^2}\right)x'^2 + z'^2\right)^3}$$

$$= \left(\frac{1}{1-v^2}\right)\frac{(x'^2+z'^2)}{\left[\frac{x'^2+(1-v^2)z'^2}{1-v^2}\right]^2} = \frac{(1-v^2)^3}{(1-v^2)}\frac{(x'^2+z'^2)}{\left[(x'^2+z'^2)\left(1-\frac{v^2 z'^2}{(x'^2+z'^2)}\right)\right]^3}$$

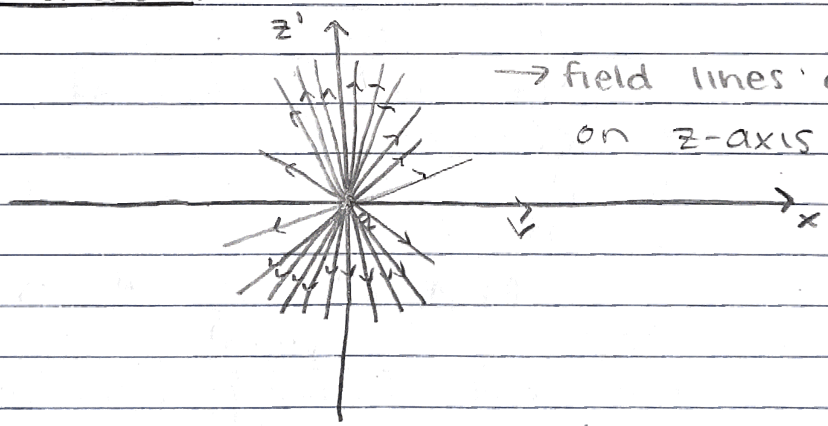
$$= \underbrace{(1-v^2)^2}\left(\frac{1}{(x'^2+z'^2)^2}\right)\underbrace{\left(\frac{1}{1-v^2\cos^2\theta'}\right)^3}$$

these determine the shape of the field

OR: $$|\vec{E}'| = \left(\frac{1}{\gamma'^2}\right)\left(\frac{1}{r'^2}\right)\left(1-v^2\cos^2\theta'\right)^{-3/2}\left(\frac{q}{4\pi\epsilon_0}\right)$$

$\Rightarrow$ lorentz contraction is occuring to the field lines

as drawn:



$\rightarrow$ field lines are concentrated on z-axis and sparse on x-axis

In the book's notation (not sphereical):

$$|\vec{E}'| = \left(\frac{1}{\gamma'^2}\right)\left(\frac{1}{r'^2}\right)\left(1-v^2\sin^2\theta\right)^{-3/2}\left(\frac{q}{4\pi\epsilon_0}\right)$$

$\rightarrow$ can a stationary charge distribution give $\vec{E}'$?
(line integral along field line)

$\oint_\gamma \vec{E}\cdot d\vec{s} \neq 0$ $\Rightarrow$ No electrostatic configuration exists

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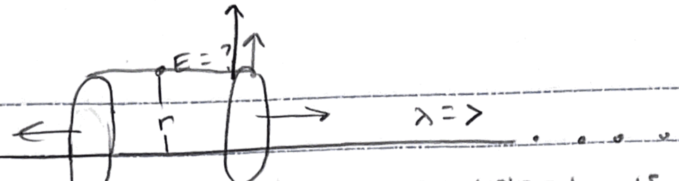
review:



$\lambda = \frac{Q}{L}$

infinite charged wire: what is E-field r distance away. → use GAUSS' LAW!

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}$$

$$E(2\pi r \ell) = \frac{\lambda \cancel{\ell}}{\epsilon_0} \qquad \boxed{E = \frac{\lambda}{2\pi r \epsilon_0}\hat{r}}$$

last time: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ — LT

$$\boxed{\begin{array}{l} E'_\perp = \gamma E_\perp \\ E'_\parallel = E_\parallel \end{array}}$$

lorentz transformations to the e-field both perpendicular and horizontally.

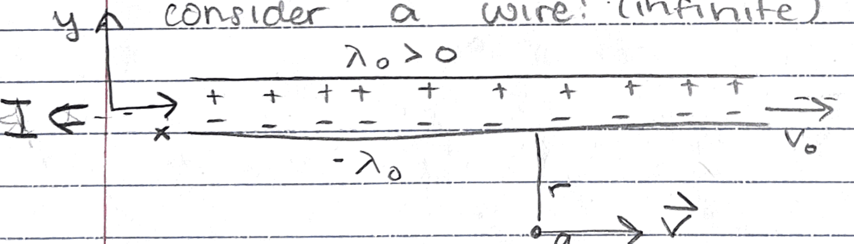
Forces: $\vec{F} = \frac{d\vec{p}}{dt} \longleftrightarrow \vec{F}' = \frac{d\vec{p}'}{dt'}$ → in ∥ direction: γp → in ⊥ & ∥ directions: $= \gamma dt$

w/ final result:

$$\boxed{\begin{array}{l} \vec{F}'_\perp = \gamma \vec{F}_\perp \\ \vec{F}'_\parallel = \vec{F}_\parallel \end{array}}$$

so if $\vec{F} = q\vec{E} \longleftrightarrow \vec{F}' = q\vec{E}'$; Then no change as we switch frames.

consider a wire: (infinite)

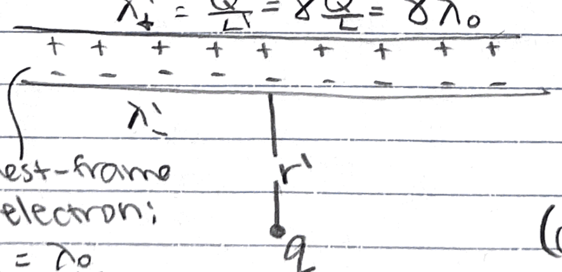


★in this frame★ (S) stationary ⊕ charges moving ⊖ charges right at speed $\vec{v}_0$ (current flowing left)

- adding charge q at distance r:

In frame S: $\vec{E} = 0$ outside wire: ? $\vec{F} = q\vec{E} = 0$?

↳ S' = rest frame of q:



in rest-frame of electron: $\lambda_- = \frac{\lambda_0}{\gamma_0}$

negative charge now move @ u:

$$\vec{u} = \frac{\vec{v}_0 - \vec{v}}{1 - \frac{v_0 v}{c^2}} =$$

(continued on next page)

$$\gamma_u = \frac{1}{\sqrt{1-u^2}} = \frac{1}{\sqrt{1-\left(\frac{v_0-v}{1-v_0v}\right)^2}} = \frac{1-v_0v}{\sqrt{1-2v_0v+v_0^2v^2-(v_0^2+v^2-2v_0v)}}$$

$$= \frac{1-v_0v}{\sqrt{1+v_0^2v^2-v_0^2-v^2}} = \frac{1-v_0v}{\sqrt{(1-v^2)(1-v_0^2)}} = \gamma_0\gamma_v(1-v_0v)$$

The total linear charge density: (in S')

$$\lambda' = \lambda_+' + \lambda_-'$$

$$= \gamma\lambda_0 - \frac{\lambda_0}{\gamma_0}\gamma_u$$

γ for e^- from rest to u

rest frame density

$$= \gamma_v\lambda_0 - \frac{\lambda_0}{\gamma_0}\gamma_0\gamma_v(1-v_0v)$$

$$= \gamma_v\lambda_0 - \lambda_0\gamma_v(1-v_0v) = \gamma_v\lambda_0(1-(1-v_0v_0))$$

$$= \boxed{\gamma_v\lambda_0 v_0 v > 0}$$

Over finite length of the wire, $q_{encl} \neq 0$

$\Rightarrow$ therefore we have a resulting e-field

↳ must be due to the magnetic field:

in S':

$$\vec{E}_{y'}' = \frac{-\lambda_0\gamma_v v_0 v}{2\pi\epsilon_0 r'}$$

$$F_{y'}' = qE_{y'}' = \frac{-q\lambda_0\gamma_v v_0 v}{2\pi\epsilon_0 r'}$$

in S:

$$F_y = \frac{-q\lambda_0 v_0 v}{2\pi\epsilon_0 r} \qquad r' = r$$

★ must be $qv\times B$

= magnetic field