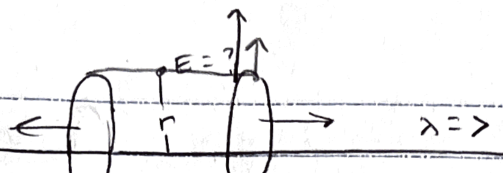
review:



3/30/23

$\lambda = \frac{Q}{L}$

infinite charged wire: what is E-field r distance away. ⟶ use GAUSS' LAW!

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0}$$

$$E(2\pi r \ell) = \frac{\lambda \cancel{\ell}}{\epsilon_0} \qquad \boxed{E = \frac{\lambda}{2\pi r \epsilon_0}\hat{r}}$$

last time: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ — LT

$$\boxed{\begin{aligned} E'_\perp &= \gamma E_\perp \\ E'_\parallel &= E_\parallel \end{aligned}}$$

lorentz transformations to the e-field both perpendicular and horizontally.

Forces: $\vec{F} = \frac{d\vec{p}}{dt} \longleftrightarrow \vec{F}' = \frac{d\vec{p}'}{dt'}$ ⟶ in ∥ direction: γp

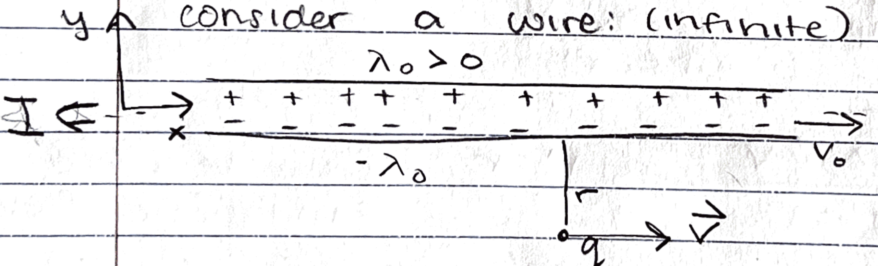
dt' ⟶ in ⊥ & ∥ directions: $= \gamma dt$

w/ final result:

$$\boxed{\begin{aligned} \vec{F}'_\perp &= \gamma \vec{F}_\perp \\ \vec{F}'_\parallel &= \vec{F}_\parallel \end{aligned}}$$

so if $\vec{F} = q\vec{E} \longleftrightarrow \vec{F}' = q\vec{E}'$; Then no change as we switch frames.

consider a wire: (infinite) ☆ in this frame ☆ (S)



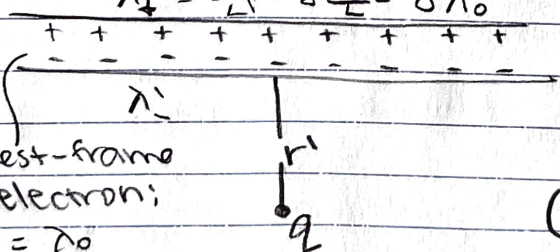
stationary ⊕ charges
moving ⊖ charges right at speed $\vec{v}_0$
(current flowing left)

- adding charge q at distance r:

In frame S: $\vec{E} = 0$ outside wire: ? $\vec{F} = q\vec{E} = 0$?

↳ S' = rest frame of q:

$\lambda'_+ = \frac{Q}{L'} = \gamma \frac{Q}{L} = \gamma \lambda_0$



in rest-frame of electron: $\lambda_- = \frac{\lambda_0}{\gamma_0}$

negative charge now move @ u:

$$\vec{u} = \frac{\vec{v}_0 - \vec{v}}{1 - \frac{v_0 v}{c^2}} =$$

(continued on next page)

$$\gamma_u = \frac{1}{\sqrt{1-u^2}} = \frac{1}{\sqrt{1-\left(\frac{V_0 - V}{1-V_0V}\right)^2}} = \frac{1-V_0V}{\sqrt{1-2V_0V+V_0^2V^2-(V_0^2+V^2-2V_0V)}}$$

$$= \frac{1-V_0V}{\sqrt{1+V_0^2V^2-V_0^2-V^2}} = \frac{1-V_0V}{\sqrt{(1-V^2)(1-V_0^2)}} = \gamma_0\gamma_v(1-V_0V)$$

The total linear charge density: (in S')

$$\lambda' = \lambda_+' + \lambda_-'$$

$$= \gamma\lambda_0 - \frac{\lambda_0}{\gamma_0}\gamma_u$$

γ for e^- from rest to u

rest frame density

$$= \gamma_V\lambda_0 - \frac{\lambda_0}{\cancel{\gamma_0}}\cancel{\gamma_0}\gamma_V(1-V_0V)$$

$$= \gamma_V\lambda_0 - \lambda_0\gamma_V(1-V_0V) = \gamma_V\lambda_0(1-(1-V_0V_0))$$

$$= \boxed{\gamma_V\lambda_0V_0V > 0}$$

Over finite length of the wire, $q_{encl} \neq 0$

$\Rightarrow$ therefore we have a <u>resulting</u> <u>e-field</u>

↳ must be due to the magnetic field:

in S':

$$\vec{E}_{y'} = \frac{-\lambda_0\gamma_V V_0V}{2\pi\epsilon_0 r'}$$

$$F_{y'} = qE_{y'} = \frac{-q\lambda_0\gamma_V V_0 V}{2\pi\epsilon_0 r'}$$

in S:

$$F_y = \frac{-q\lambda_0V_0V}{2\pi\epsilon_0 r} \qquad r' = r$$

★ must be $qV\times B$

= magnetic field

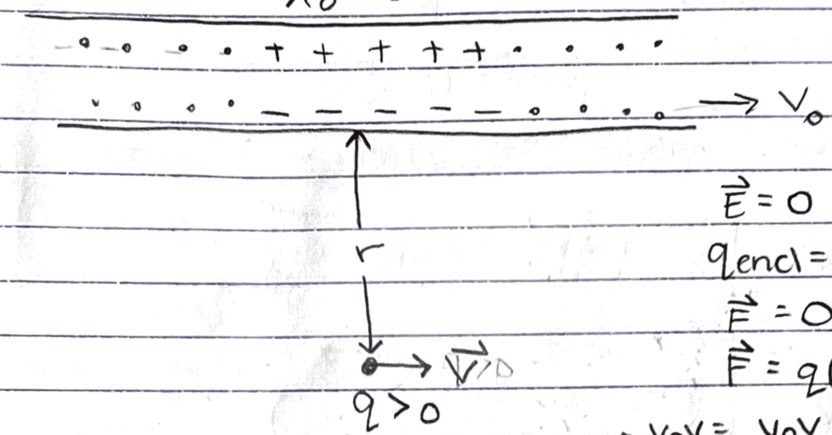
4/3/23

Gedanken experiment continued

in frame S:

$\lambda_0 > 0$

$\lambda_- = \frac{\lambda_0}{\gamma_0}$



$\vec{E} = 0$

$q_{encl} = 0$

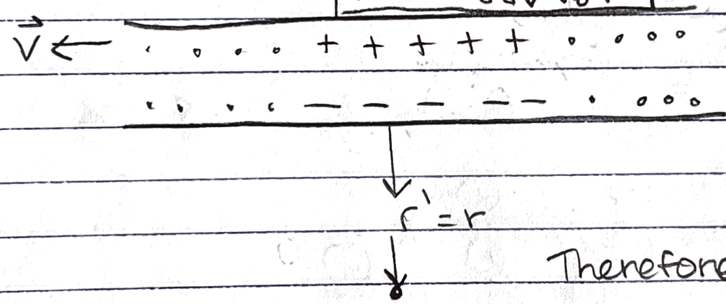
$\vec{F} = 0$

$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$

★ $V_0V = \frac{V_0V}{c^2}$ (we just dropped c^2 for calculations!)

In frame S':

$\boxed{\lambda' = \lambda_0 \gamma_v V_0 V}$ → There IS an E-field



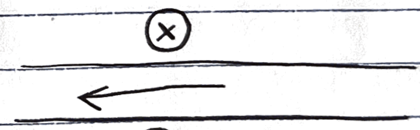
$\vec{E}' \neq 0 \rightarrow \vec{E} = y$-direction

$E'_y = -\frac{\lambda^0 \gamma_v V_0 V}{2\pi\epsilon_0 r'}$ (down)

Therefore: $\vec{F}' = \frac{-q\lambda_0\gamma_v V_0 V}{2\pi\epsilon_0 r'} \hat{j}' \neq 0$

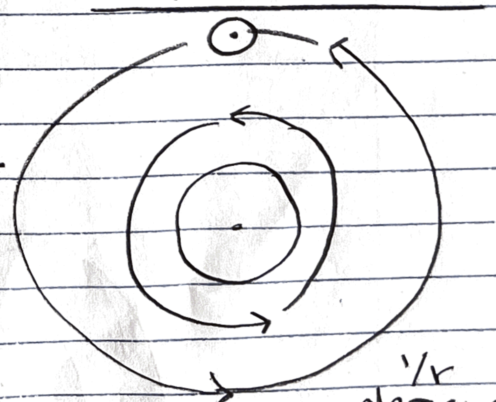
$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$ $\vec{B}$: ⊙ $F'_\perp = \gamma F_\perp$

$\Rightarrow \vec{B} = \frac{V_0\lambda_0}{2\pi\epsilon_0 c^2 r}$ (out)



$V_0\lambda_0 = I$

$\Rightarrow \boxed{\vec{B} = \frac{I}{2\pi\epsilon_0 c^2 r}}$



$\epsilon_0 c^2 = \frac{1}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7} \frac{kgm}{c^2}$

★New right-hand rule:
thumb in direction of current → fingers in curl of magnetic field

1/r dependance

→ ☆ $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ (we will come back to this when we find light) in same direction so: (similar to GAUSS' LAW)

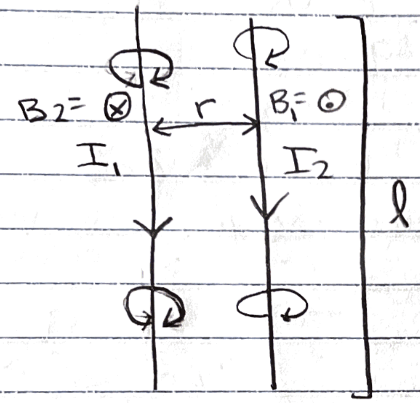
$$\oint \vec{B} \cdot d\vec{l} = B \oint dl = B(2\pi r) = \mu_0 I_{encl}$$

$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$$

ampère's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

returning to car-cable current demo:

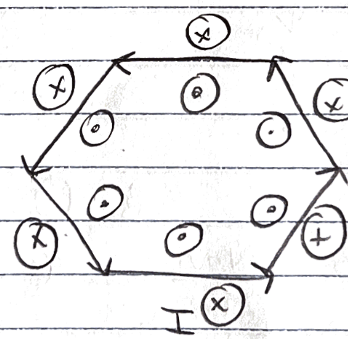


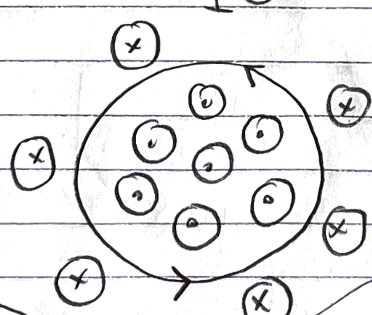
$$|\vec{B}_1| = \frac{\mu_0 I_1}{2\pi r}$$

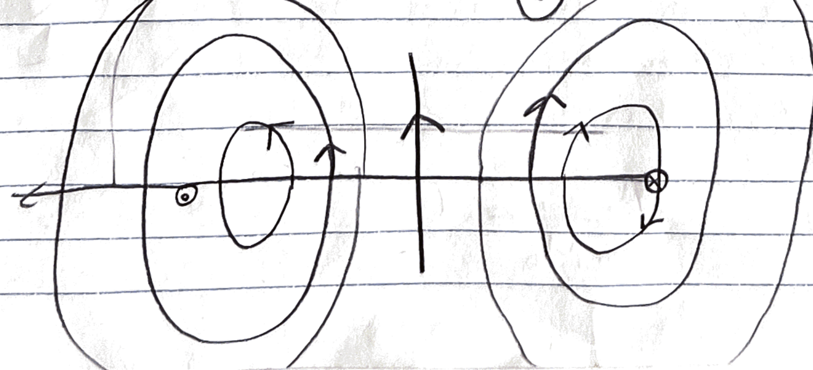
$\vec{F} = I_2 \vec{l}_2 \times \vec{B}_1$ $\vec{l}$ points in I direction

$$= I_2 l B_1 = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

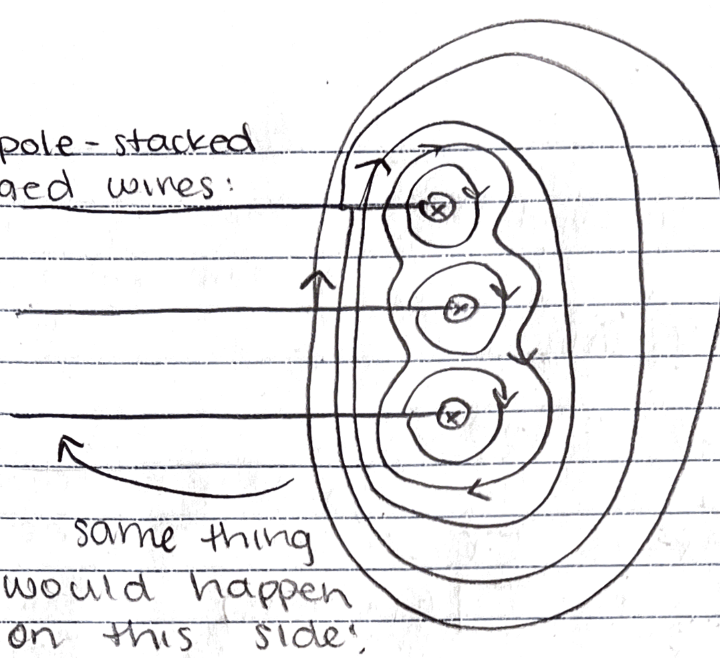
$\Rightarrow$ what is $\vec{B}$ for a bent wire or loop? yes!







multipole - stacked charged wires:



(sketch - field lines seperate further w/more distance)

same thing would happen on this side!

Last time: $\vec{B}$-field of a wire: 4/5/23

$$B = \frac{\mu_0 I}{2\pi r}$$

→ moving charges source the $\vec{B}$-field (as far as we know these are magnetic charges)

$\vec{\nabla} \cdot \vec{B} = 0$

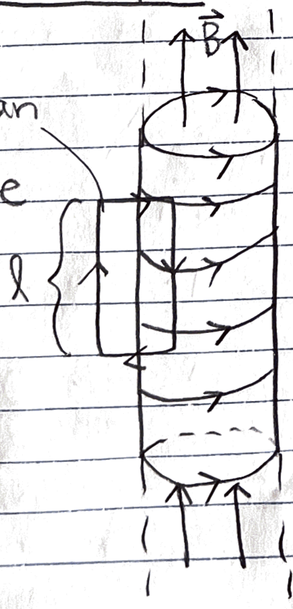
$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ ← charge density

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$$ Ampere's law

today: example + more local form of ampere's law

example: long solenoid:

amperian loop = rectangle



- lots of wire loops w/current I
- n-turns per unit length

→ B-field lines go straight up

⇒ $\vec{B}$ outside vanishes, $\vec{B}$ inside goes upward

⇒ let's find the $\vec{B}$ inside and out:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

(continued on next page)

4/5/23 '23

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

* n= number of coils

$\int_I B\,dl = \mu_0 I n l$

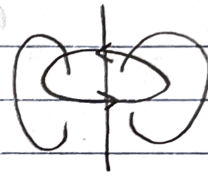
$B\int_I dl = \mu_0 n I l$

easy to create desired magnetic field w/solenoid

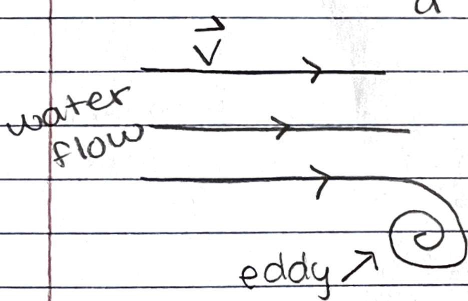
$B\cancel{l} = \mu_0 n I \cancel{l}$

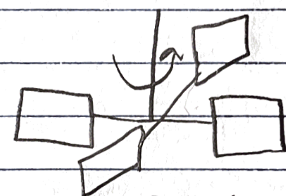
$\boxed{B = \mu_0 n I}$ (upwards)

★ $\vec{B}$-inside vanishes
★ $\vec{B}$-outside goes up



example 2: we are examining a brook:





(badly-drawn paddle wheel)

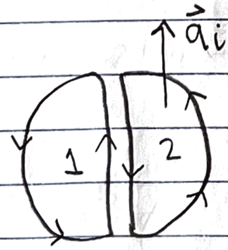
→ placed to find speed of spin of eddy

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ (ampere's law)

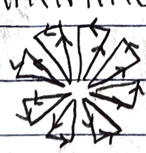
defining the "circulation" (spin) as:

$\Gamma = \oint_{loop} \vec{v}\cdot d\vec{l}$ ⇒ we would like a local version of this!

$\vec{a}_i = a_i \hat{n}_i$



→ subdivide infinitley



$\Gamma_{loop} = \Gamma_1 + \Gamma_2$

into "n"-loops $n \to \infty$

$\Gamma = \sum_{i=1}^{N} \Gamma_i$

$\Gamma_i = \oint_i \vec{v}\cdot d\vec{l}$

Idea: multiply by 1!

$\Gamma_i \xrightarrow[n\to\infty]{} 0$

= useless, we want something finite

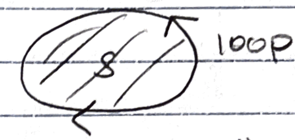
(continued on next page)

example 2 continued: 4/5/23

$$T = \sum_i \frac{a_i}{a_i} \oint_i \vec{v}_i \cdot d\vec{l}$$ → this could be finite

$$T = \sum_{i=1}^{N} \vec{\nabla} \times \vec{v} \cdot \hat{n}_i (a_i)$$

$$\boxed{T = \int_S (\vec{\nabla} \times \vec{v}) \cdot d\vec{a}}$$



surface "S" = enclosed by loop

STOKE'S THRM!! (3D divergence)

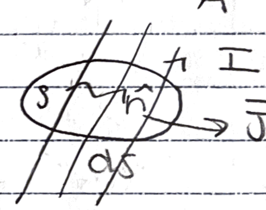
$$\boxed{\int_S (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint_{\partial S} \vec{v} \cdot d\vec{l}}$$ → divergence thm:

$$\int_V (\nabla \cdot E) dx^3 = \int_{\partial V} \vec{E} \cdot d\vec{a}$$

$$\int_S \vec{\nabla} \times \vec{B} \cdot d\vec{a}$$

$$= \mu_0 \underbrace{\int_S \vec{J} \cdot d\vec{a}}_{I_{enclosed}}$$

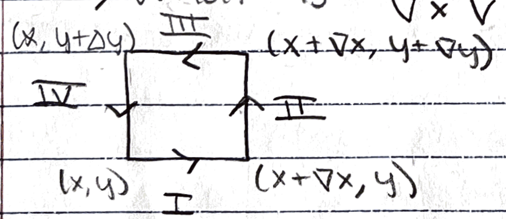
current density: $\vec{J} = \frac{I \hat{n}}{A}$

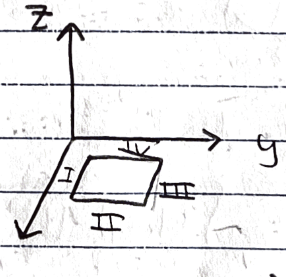


$$\Rightarrow \int_S (\vec{\nabla} \times \vec{B} - \mu_0 \vec{J}) \cdot d\vec{a} = 0$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

⟹ what is $\vec{\nabla} \times \vec{V}$? (textbook chapter 2)





$$\oint \vec{v} \cdot d\vec{l} = \int_{I} \vec{v} \cdot d\vec{l} + \int_{II} \vec{v} \cdot d\vec{l} + \int_{III} \vec{v} \cdot d\vec{l} + \int_{IV} \vec{v} \cdot d\vec{l}$$

small loop:

$$\int_{I} v \cdot d\vec{l} = \int_{I} V_x dx$$ ← taylor series:

$$\int \Big(\underset{\uparrow \text{const}}{V_x(x,y)} + \underset{\uparrow \text{const}}{\frac{\partial V_x}{\partial x}} \Delta x \Big) dx$$

$$= V_x(x,y)\Delta x + \frac{\partial V_x}{\partial x} \Delta x^2$$

along side I

(cont. on next page)

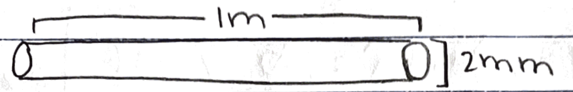
along side II

$$\int_{\mathrm{II}} \vec{v}\cdot d\vec{l} = \int_{y}^{y+\Delta y} v_y\,dy \overset{\text{taylor series:}}{\backsimeq} \int_{y}^{y+dy}\left(v_y(x,y) + \frac{\partial v_y}{\partial x}\Delta x + \frac{\partial v_y}{\partial y}\Delta y\right)dy$$

$$= v_y\Delta y + \frac{\partial v_y}{\partial x}\Delta x\Delta y + \frac{\partial v_y}{\partial y}\Delta y^2$$

$\boxed{\text{Brian lecture}}$ 4/7/23

example: 1m length wire, 1-2mm diameter



volume $= \pi\times10^{-6}\,m^3$

$\rho = 8.5\times10^{22}\,e^-/m^3$

in wire $= 2.6\times10^{17}e^- = \boxed{4.16\times10^{-2}C}$

→ in car-wire demo:

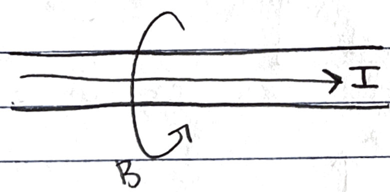
10 amps flowing through wire

$\uparrow \times n = 10C$

$\Rightarrow \dfrac{10\,amp}{C\cdot s}$

$n \sim 250$ → e^-'s moving 250m/s through wire

⇒ doesn't take relativistic velocities for magnetism to show up, even though it is a relativistic effect.



$B = \dfrac{\mu_0 I}{2\pi}$ (ampere's law for long wires)

$$\oint_{\Gamma} \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} \quad \text{(ampere's law)}$$

$$= \int_S \mu_0 \vec{J}\cdot d\vec{a}$$

differential area

current density

Γ: closed loop over some path

S: surface bound by path

Stoke's theorem: $\underbrace{\int_S (\nabla\times B)\cdot da = \int_{\Gamma} B\cdot dl}_{\text{relation between area and line around}}$

→ cont. on next page

4/7/23

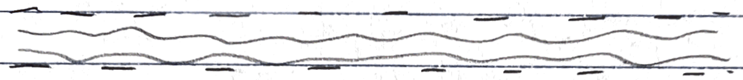
Stokes theorem appli to ampere's law:

$$\int_S \text{curl} B \cdot d\vec{a} = \int_S \mu_0 J \cdot da$$ ⇒ if integrals are equal without equal bounds: integrands = equal

$$\nabla \times B = \mu_0 J$$

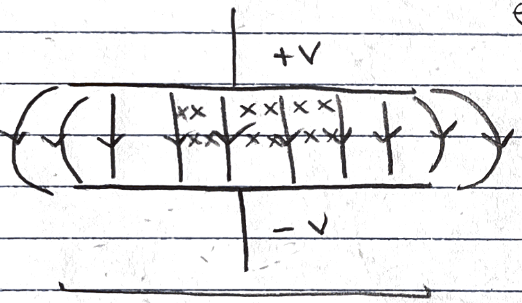
→ talk abt. Brian + Gordon's experiment (ACORN)

one hole = one large bumb



lots of holes = more uniform magnetic field

capacitor (harder to manipulate e-field)



$$\nabla^2 V = 0$$ (Laplace's equation)

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} = 0$$

points spread evenly through material

↳ to fix problem: put more points at points of interest (edges of capacitor)