Electromagnetism (PHYS 295): Solutions 6

Spring 2025 v1.0

Solutions:

(1) Solution in text

(2) Solution in text

(3) Starting an engine:

(a) From the diagram and choices of the current directions, $I_3 = I_1 + I_2$, the junction condition. For loops I used outside loop, obtaining

$$-I_1R_C - I_3R_S - I_1R_C - I_1R_1 + \mathcal{E}_1 = 0 \text{ or } 12.5 - 0.025I_1 - 0.15I_3 = 0$$

From the lower loop one has the equation

$$-I_3R_S - I_2R_2 + \mathcal{E}_2 = 0 \text{ or } 10.1 - 0.15I_3 - 0.10I_2 = 0$$

(b) Multiplying the first equation by 0.025 and the second by 0.10 and adding gives

$$0.2525 + 1.25 - 0.0025(I_1 + I_2) - 0.01875I_3 = 0.$$

Using the junction condition gives $I_3 = 71$ A .

(c) $I_3 = 71$ A > 60 A, so "yes!" it will turn over.

(4) Light bulbs

(a) If bulb 1 is twice as bright as bulb 2 then it is consuming twice the power so $P_{1P} = 2P_{2P}$. (The "P" is for parallel.) The two bulbs are in parallel and so have the same potential drop,

$$\mathcal{E} = I_1R_1 = I_2R_2 \text{ so } \frac{I_1}{I_2} = \frac{R_2}{R_1}.$$

Since power goes as $P = I^2R$, the relative brightness means that

$$I_1^2R_1 = 2I_2^2R_2 \implies \frac{I_1^2}{I_2^2} = 2\frac{R_2}{R_1}.$$

Using the above relation we have

$$\frac{I_1^2}{I_2^2} = \frac{R_2^2}{R_1^2} = 2\frac{R_2}{R_1} \text{ or } R_2 = 2R_1.$$

Bulb 2 has twice the resistance of bulb 1. Note that the currents in this case the currents satisfy

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} = 2 \implies I_2 = \frac{I_1}{2}$$

which will be useful in part (b).

(b) Now the resistors are in series so

$$R_{equiv} = R_1 + R_2 = 3R_1.$$

Since $\mathcal{E} = IR_{equiv} = 3IR_1$, the current passing through each bulb is $I = \mathcal{E}/3R_1$. The powers are

$$P_{1S} = I^2R_1 = \frac{\mathcal{E}}{9R_1} \text{ and } P_{2S} = I^2R_2 = \frac{\mathcal{E}R_2}{9R_1} = \frac{2}{9}\frac{\mathcal{E}}{R_1}.$$

Now bulb 2 is twice as bright. To compare to the parallel case, I'll use the relation $I_1 = \mathcal{E}/R_1$. The power through bulb 1 in parallel is

$$P_{1P} = I_1^2 R_1 = \frac{\mathcal{E}^2}{R_1} = 9P_{1S}$$

The parallel case is nine times brighter than series case. As for bulb 2,

$$P_{2P} = I_2^2 R_2 = \frac{I_1^2}{4} R_2 = \frac{\mathcal{E}^2}{2R_1} = \frac{9}{2} P_{2S}$$

So bulb 2 in the parallel configuration is 9/2 times brighter than the series case.

(5) Volcanoes! It is useful to have the factor γ. With $v = 4/5\, c$ then

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \frac{16}{25}}} = \frac{5}{3}.$$

(a) Moving object shrink so the distance between the mountains in the spacecraft's frame is

$$L_S = \frac{500\ \text{km}}{\gamma} = 300\ \text{km}.$$

I have labeled this "L_S" in the sketch in part (5c).

(b) There are a number of ways you could solve for this time delay. I'll give two solutions. First, from the diagram in part (5c) we can see that

$$D = v\Delta T + L_S$$

where L_S is the distance between the mountains in the spacecraft frame and D is the distance between the eruptions in the spacecraft's frame. The slip in simultaneity gives

$$\Delta T = \frac{vD}{c^2} \text{ or } D = \frac{c^2 \Delta T}{v}.$$

Equating these two expressions for D and collecting terms gives

$$\left(\frac{c^2}{v}\right)\left(1 - \frac{v^2}{c^2}\right)\Delta T = L_S.$$

Recognizing γ and solving for ΔT gives

$$\Delta T = \frac{v\gamma^2 L_S}{c^2}.$$

So, denoting the distance between the mountains in their own frame with L_M,

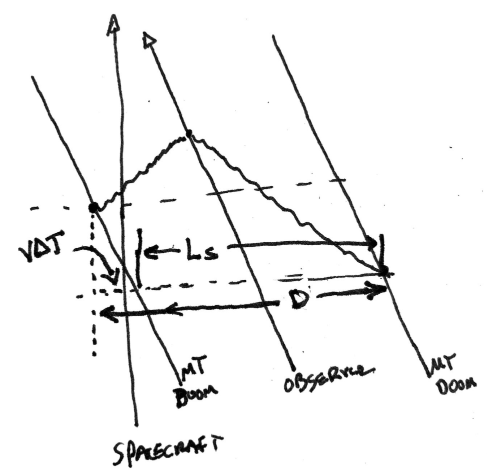
$$\Delta T = \frac{v\gamma L_M}{c^2} = \frac{\frac{4}{5}c\frac{5}{3}500\ \text{km}}{c^2} = \frac{2}{9} \times 10^{-2}\ \text{s} \simeq 2.2\ \text{ms}.$$

A second way to solve this is to use Lorentz transformations.

$$\Delta t = \gamma\left(0 + v\Delta x'/c^2\right) = \frac{\gamma v \Delta x'}{c^2},$$

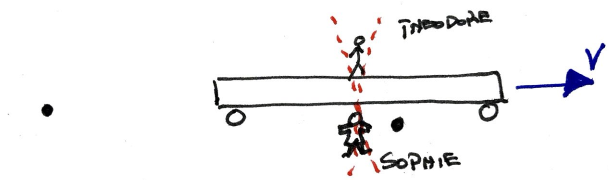
which is the same as above since $\Delta x' = L_M$.

(c) Here's a sketch in the spacecraft's frame



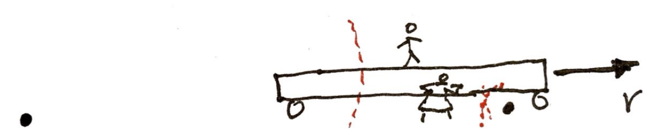
(6) (2 pts.) The super-fast WorldStar train

(a) I found it helpful to draw these "snapshot" diagrams with the analogous spacetime diagrams, which I include at the end. In Sophie's reference frame the WorldStar train passes by, say moving to the right. Here's the moment when the light fronts reach Sophie.



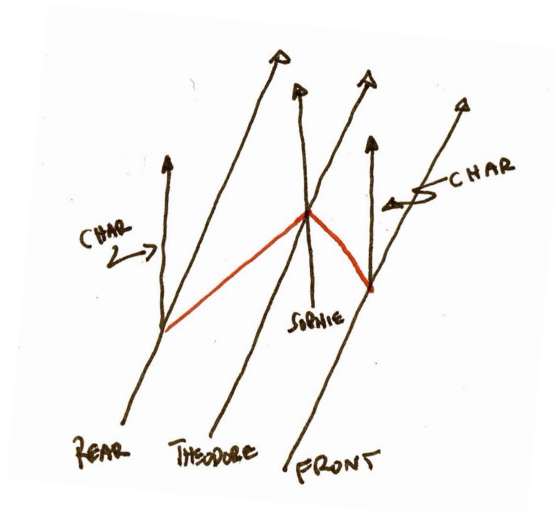
Key elements include light fronts arriving at Sophie and Theodore and the char marks on the ground displaced to the rear since the light took some time to arrive while the trains moved.

Here's the situation shortly after the lighting strikes.

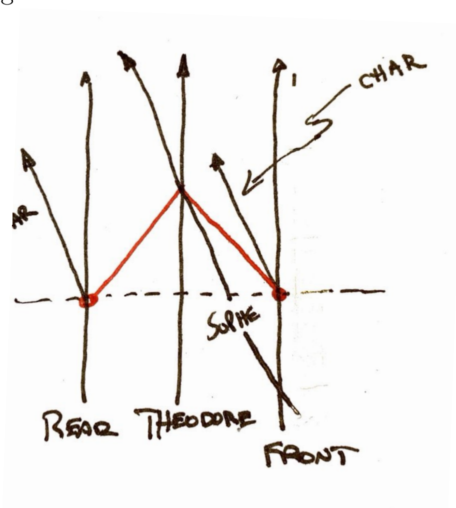


Notice how the light fronts are equidistant from Sophie but are of different distances from the char marks. This occurs because the events are not simultaneous in Sophie's frame.

Here's the spacetime diagram of the events in Sophie's frame



(b) Here's the spacetime diagram of the events in Theodore's frame



(c) In Sophie's frame the rear of the train in struck by lighting first. The front is struck next. Finally the flashes of light (or light fronts) are seen by Sophie.

(d) In Theodore's frame the light strikes both ends of the train simultaneously and then Theodore sees the flashes of light, which is also the same event when Sophie sees the flashes.

(7) The earth has a radius of about 6400 km. So light, or a radio signal, traveling around the Earth would take $\pi r/c \simeq \pi(6400\text{ km})/3\times10^8 \simeq 0.067$ s. (Even if it passed through the earth somehow, light would take $d/c \simeq 0.04$ s to make the trip.) Thus, if the signal is light-like it could not take less time than this. So there is reason to be skeptical about the 'less than one-hundredth of a second' claim since a signal could not reach the mind reader in that amount of time.

(8) The 'ladder in barn' situation with the speed of the runner at 0.866c and both proper lengths are 5 m.

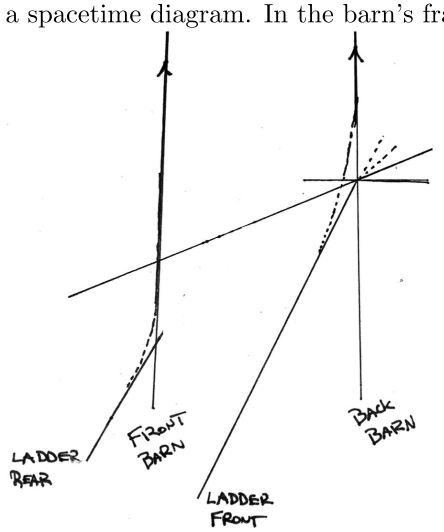
(a) We'll need γ so

$$\gamma = \frac{1}{\sqrt{1-(0.866)^2}} \simeq 2$$

Use length contraction to obtain $L' = L/\gamma = 5/2 = 2.5$ m for the length of the ladder in the barn's frame. Likewise, the length of the barn in the ladder's frame is $5/2 = 2.5$ m

(b) If the runner arranges to come to a stop when the front of the ladder reaches the back of the barn in the proper frame of the runner then the ladder does not fit in the barn. When the ladder starts to slow down only one half of its length is in the barn. As it slows the barn expands and, in the end, both the ladder and barn have the same length.

(c) It may be easiest to see in a spacetime diagram. In the barn's frame,



The dashed lines show the ladder during its deceleration. The history in the barns frame is: The front of the ladder arrives and then the rear the ladder nears the front of the barn as the ladder slows. As it comes to rest both barn and ladder have the same length.