Electromagnetism (PHYS 295): Solutions 7

Spring 2025 v1.1

Solutions:

(1) K and signals

(a) By definition $T' = KT$. Since the frequency is inversely proportional to the period we have that the observed frequency $f_o = 1/T_o = 1/T'$ must be related to the emitted frequency $f_e = 1/T_e = 1/T$ via,

$$f_o = \frac{1}{K} f_e.$$

(b) Similarly since the wavelength is inversely related to the frequency $c = f\lambda$ we see that the redshift

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{cT_o - cT_e}{cT_e} = K - 1 = \sqrt{\frac{1 + v/c}{1 - v/c}} - 1.$$

There are other ways to express this such as $\gamma(1 + v) - 1$.

(2) Velocity addition revisited: Starting from the product relation

$$K_{AC} = K_{AB} K_{BC}$$

and squaring we find

$$\frac{1 + v_{AC}}{1 - v_{AC}} = \frac{1 + v_{AB}}{1 - v_{AB}} \frac{1 + v_{BC}}{1 - v_{BC}}.$$

(I am setting $c = 1$ here to save writing.) Multiplying by the denominators gives

$$(1 + v_{AC})\,(1 - v_{AB})\,(1 - v_{BC}) = (1 + v_{AB})\,(1 + v_{BC})\,(1 - v_{AC})\,.$$

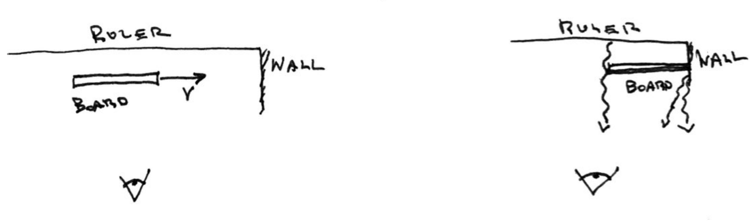
Multiplying this all out and canceling and gathering terms, this reduces to

$$v_{AC}\,(1 + v_{AB} v_{BC}) = v_{AB} + v_{BC}$$

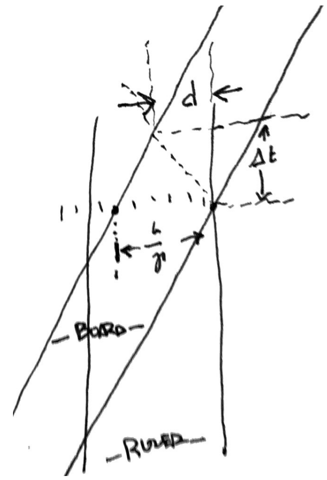
which gives the expected

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + v_{AB} v_{BC}}.$$

(3) Here's a "top view" sketch of the situation:



Although I have drawn some separation between the board and ruler, I will assume they are very close. The eye of the observer is shown. Since we are asked about this frame then this is the frame we should use to analyze the situation. Here's a spacetime diagram of the same scenario



In the observer's frame, the board is moving and so contracted to length L/γ. But what does the observer see? Well, let's make the assumption that the board is like a spring. If so, the signal to stop will not reach the trailing end until light can reach it. This is after a time Δt and the light will have traveled $d = c\Delta t$. We can also express this as

$$d = \frac{L}{\gamma} - v\Delta t$$

as shown in the spacetime diagram. Thus,

$$d = \frac{L}{\gamma} - v\frac{d}{c} \text{ and so the distance is } d = \frac{L}{\gamma(1+v/c)} = L\sqrt{\frac{1-v/c}{1+v/c}}.$$

This is the nearest-to-wall ruler marking that would be (in theory) visible. At relativistic speeds, however, the collision of the board with the wall would produce a shower of high energy particles that would likely obscure the ruler. Ah well.

(4) Sunlight:

(a) By conservation of energy for the process $p + D \to {}^3He + \gamma$ the energy of the photon is

$$E_\gamma \simeq (1.6724 + 3.3432 - 5.0058) \times 10^{-27} c^2 \simeq 8.82 \times 10^{-13} \text{ J } \simeq 55 \text{ MeV},$$

(b) This lies in the gamma ray part of the spectrum. BTW the wavelength is about 2.26×10^{-13} m and the frequency is about 1.3×10^{21} Hz.

(c) If the earth is bathed with 1350 W/m^2 then so is every other portion of a sphere with the (average) Earth's orbital radius. This means that the sun produces

$$P = 4\pi r^2 I \simeq 3.82 \times 10^{26} \text{ W}$$

of power. That's one bright bulb. (I used a radius of 1.5×10^8 km.) Using $E = mc^2$ to convert to mass gives about 4.2×10^9 kg per second, or 4.2×10^6 metric tons per second.

(d) Since

$$P = \frac{dE}{dt} = \frac{dM}{dt}c^2$$

leaving the sun, the lifetime τ is determined by

$$\tau = -\frac{1}{P}\int_M^0 dM = \frac{M}{P} \simeq 4.76 \times 10^{20} \text{ s } \simeq 1.5 \times 10^{13} \text{ yrs}$$

which is *way* too long, partly because the sun will end in another (white dwarf) star, which about half as massive. Also, the rate changes as the fusion process changes.

(5) (In this solution I mostly set $c = 1$ so $v = \beta$.) If γ equals 1.2 then the speed is

$$v = \sqrt{1 - 1/\gamma^2} \simeq 0.553$$

when $c = 1$ or $v \simeq 0.553c$. In the same "lab" frame when the test charge moves at v, the electrons move at $v_o = 0.8$ so in the proper frame of the test charge (the "prime frame"),

$$v_0' = \frac{v_o - v}{1 - v_o\, v} \simeq 0.443.$$

The relative sign can be gleaned from Fig. 5.22(a) where both v and v_o point to the right so in the prime frame the speed is reduced - subtracted.

The charge density in the prime frame is

$$\lambda' = \gamma v v_o \lambda_o \simeq 0.531\lambda_o$$

from equation (5.24) (and also in class). Hence, the relative charge density is $\lambda'/\lambda_o = 0.531$.

(6) This is a mass spectrometer

(a) For positive particles to curve that way under the $qv \times B$ force, the magnetic field must point "up" or out of the page.

(b) From mechanics, masses moving on a circular trajectory satisfy

$$a = \frac{v^2}{r} \text{ and since } F = qvB = ma$$

we have

$$\frac{q}{m} = \frac{v}{Br}$$

(so far). How to remove the speed? By conservation of energy

$$\frac{1}{2}mv^2 = q\Delta V \text{ so } v = \sqrt{2q\Delta V/m}$$

So when we square and substitute we have

$$\left(\frac{q}{m}\right)^2 = \frac{2q\Delta V}{mB^2r^2} \text{ or } \frac{q}{m} = \frac{2\Delta V}{B^2r^2}$$

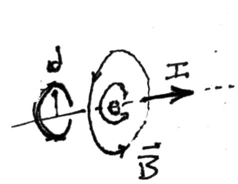
as expected.

(c) There is a clever way to solve this directly but computing the radii works:

$$r = \sqrt{\frac{2\Delta V m}{qB^2}}$$

for both isotopes, finding 4.94 m for the 68 and 5.01 for the 70. The difference yields $\Delta r \simeq 7.2$ cm but this asks for the separation or $2\Delta r \simeq 14.4$ cm.

(7) Finding **B**-fields I: Here's a sketch of the current carrying wire, the B-field and the loop



Using the circle of radius of radius d in Amper's law,

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o I_{encl},$$

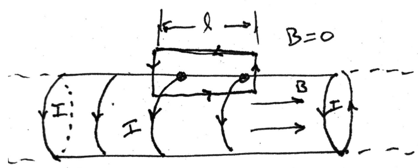
the integral is $\int B ds = B \int ds$ since the path follows the magnetic field around the wire so

$$B 2\pi d = \mu_o I.$$

Solving for the magnetic field gives

$$B = \frac{\mu_o I}{2\pi d}.$$

(8) Finding **B**-fields II: Here's a sketch of the solenoid, the B-field and the loop



Far from the ends of the solenoid the magnetic field will be constant inside. For the outside note that we have current coming out of the page at the top and into the page at the bottom. By the right hand rule and superposition, the magnetic field vanishes (outside).

Using the loop of length ℓ in Amper's law,

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o I_{encl},$$

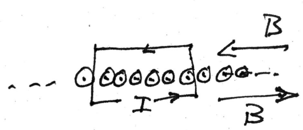
the integral is $B\ell$ since the path follows the magnetic field inside so

$$B\ell = \mu_o n I \, \ell.$$

Canceling the factor of ℓ, we have

$$B = \mu_o n I.$$

(9) Finding **B**-fields III: : Here's a sketch of the solenoid, the B-field and the loop



By eth right hand rule, on the top the magnetic field points to the left. Below the sheet it points to the right. Using the loop of length ℓ in Amper's law,

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o I_{encl},$$

the integral is $2B\ell$ since the path follows the magnetic field inside so

$$2B\ell = \mu_o \ell K.$$

Canceling the factor of ℓ, we have

$$B = \frac{\mu_o K}{2}.$$