Math Methods (PHYS 320):14

Spring 2019

Reading: For more on Sturm-Louville theory see Arfkin and Weber posted on the 320 website
Boas on (your) special function (if none read Ch 12 section 22)
Boas Chapter 7 section 12

(1) Is Bessel's equation self-adjoint? If so, explain. If not, how can we make it self-adjoint?

(2) Boas pg 384 problem 11

Choose one of the following two problems to solve.

(3) A remarkable test of "local realism" in quantum mechanics, proposed by Hardy in 1993, was completed in 1999. In the experiment particles pass through detectors that have two settings, a or b, and two possible outcomes for each setting, $+$ or $-$. The quantum state of a particle can described by the kets $|\,a+\rangle$, $|\,a-\rangle$, $|\,b+\rangle$, or $|\,b-\rangle$. Assume that the $+$ and $-$ basis vectors are orthonormal in each basis a or b. The state $|\,b+\rangle$ is related to the states in the a basis by

$$|\,b+\rangle = \cos\theta/2\,|\,a+\rangle + \sin\theta/2\,|\,a-\rangle\,.$$

In Dirac notation the states for more than one particle are "pasted" together. For example suppose that the first particle is in state $|\,a+\rangle_1$ and the second is in state $|\,b+\rangle_2$ then the state of the two particles is the product

$$|\,a+,b+\rangle = |\,a+\rangle_1\,|\,b+\rangle_2.$$

Quantum mechanics is unique among physical theories for allowing a superposition of such states.

The Hardy state is given by

$$|\,\psi\rangle = \cos^2\left(\frac{\theta}{2}\right)\left[\cos\left(\frac{\theta}{2}\right)|\,a+,a+\rangle - |\,a+,a-\rangle - |\,a-,a+\rangle\right]$$

where $\theta \simeq 76.35^o$. Let's check some predictions arising from the state:

(a) Show that a system prepared in the Hardy state is never found in the state $|\,a-,a-\rangle$. You can do this by checking $\langle a-,a-\,|\,\psi\rangle = 0$.

(b) Show that a system prepared in the Hardy state is never found in the state $|\,a+,b+\rangle$.

(c) Show that a system prepared in the Hardy state is never found in the state $|\,b+,a+\rangle$.

(d) Show that a system prepared in the Hardy state is in the state $|\,b+,b+\rangle$ 9 % of the time, i.e. show that $|\langle b+,b+\,|\,\psi\rangle|^2$ is approximately 0.09.

(e) EXTRA: Show that these results are incompatible with local realism. You can accomplish this by showing that the above results are impossible if you assign each particle properties for each setting. In the language of Don Spector, who spoke here some years ago, there are no "tickets" that you can give to the particles that lead to the experimentally-tested predictions above.

(4) The differential equation for a pendulum is

$$\theta'' + \omega_o^2\sin(\theta) = 0$$

where θ is the angle between the pendulum bob's position and the bob's position at rest. (While I wrote θ'' to conform to our in class notation it is often written with $\ddot{\theta}$.)

(a) In Mathematica find the numerical solution for this equation when

$$\omega_o = 2\pi \text{ and for IC's } \theta(0) = 0.02 \text{ and } \theta'(0) = 0$$

What is the period of oscillation T? You can find this from a plot of your solution.

(b) By re-running the simulation explore the relation between the starting angular position $\theta(0) = \theta_o$ and the period. Graph the period T vs. θ_o.

(c) From your explorations explain the divergence as $\theta_o \to \pi$.

(d) EXTRA: See if you can find a pretty good match for the function $T = T(\theta_o)$.