General Relativity or "GR" (PHYS 325): PS 1

Spring 2024 v1.6

As he pondered the long futile fight
To make Galileo's world right,
In a new variation
Of the old transformation,
It was Einstein who first saw the light.
- D. Morin.

This first material is on special relativity with an emphasis on the foundations of Lorentz transformations and the geometry of flat spacetime. New notation, diagrams, methods of calculating, and concepts abound!

All numbered problems are from Schutz. The chapter is given first, then the problem or 'exercise' in Schutz's lingo, e.g. 1.13 is the 13th exercise of Chapter 1.

Reading:

Chapter 1 in Schutz.
Section 2.1 on vectors
Other useful resources include Einstein's 1905 SR paper (there's a link on this page), Ellis and Williams, Mermin, and Rindler.

Problems: Due Thursday, Jan. 25 at the beginning of class.

(1) Work out the following in $c = 1$ units. (See 1.1 for a worked example.)
  (a) 1 J (You can see why we might not use this "c=1" set of units in intro lab!)
  (b) 1 eV ("electronvolt" - an energy often used in atomic and particle physics)
  (c) 1 kWh
  (d) 1 atm = 10^5 N m^{-2} (Ground level atmospheric pressure)
  (e) Choose one of the energy units above and write an explanation of what it is in everyday language.

(2) Similar to what we discussed in class, two volcanoes, Mt. Boom and Mt. Doom, are 700 km apart in their rest frame. Suppose that each erupts in a burst of light. An observer in a lab halfway between the two volcanoes receives the light from the two eruptions at the same time. The above objects (mountains, observer, and lab) are at rest with respect to each other. We found that in this frame all observers agree that the eruptions occurred simultaneously. (If you don't agree, let's talk.)

A spacecraft flying by at 4/5 c from Mt. Boom to Mt. Doom is 150 km from Mt. Boom and 550 km from Mt. Doom when they erupt, in the mountains' frame.
  (a) According to an observer on the spacecraft does the eruption at Mt. Boom occur before, at the same time, or after the eruption at Mt. Doom?
  (b) Carefully draw a space-time diagram of the events in the spacecraft's frame.
  (c) If you found that the eruptions are not simultaneous in the spacecraft's frame then compute the time delay between the two eruptions. If you found the events were simultaneous, compute the location of the spacecraft when the eruptions occurred.
  (d) What would change if the spacecraft was going the other way, i e. from Mt. Doom to Mt. Boom?

Assume the mountains and observers are all on a single line. You can also neglect any non-inertial effects due to being on the surface of the Earth.

(3) MODIFIED 1.3: Use light-seconds instead of meters and complete a spacetime diagram with parts (a), (c), (d), (i), and (l) only. A "Locus" is defined by those points or events that satisfy the given condition.

(4) 1.10 'ID'ing intervals

(5) A swift train car of length $L = 25$ m (as measured in your frame) has two synchronized clocks, one on each end. (We can suppose that the clocks were synchronized together in the middle of the train and then carried to each end in an identical manner.) Suppose that the train car moves at $v = (4/5)$c in your frame. Your assistant passes the front clock when it reads noon (12:00). You pass the rear clock at the same time in your frame.
   (a) What time does that clock read?
   (b) Draw a spacetime diagram of the history including surfaces of simultaneity for you when you read the clock and for the clocks on the train at noon.
   (c) Check that this effect is *quite* small for everyday fast trains (e.g. traveling at 250 km/hr).

(6) On transverse contraction: Consider a pair of observers and meter sticks. One pair is at rest and the other pair is moving along an axis perpendicular to the first meter stick and perpendicular to its own length. Assume that the observers are in the middle of the meter sticks.
   (a) Argue that the symmetry about the observers' shared x-axis implies both observers will see the ends of the meter sticks cross simultaneously and both observers will therefore agree if one meter stick is longer than the other.
   (b) Argue from the relativity principle that the lengths cannot be different.

(7) **Not required!** (But I'd love to hear your thoughts) In addition to the principle of relativity, suppose that there is an invariant energy scale E_* and that the speed of light depends on the energy,

$$c(E) = c\left(1 - \frac{E}{E_*}\right),$$

where the photon's energy is $E < E_*$. What are new the Lorentz transformations? Determine the physical consequences including time dilation and length contraction.