General Relativity (PHYS 325) PS 12

Spring 2024 v1.5

This week we consider sources of gravitational waves before moving on to our last application of Einstein's equations - cosmology.

Reading:

- Schutz Chapter 12 sections 1 - 2 and 12.4 (You may enjoy reading the rest of the chapter for more on GW observations.)
- Schutz Chapter 13 on cosmology - we won't be able to discuss all this but it is all good stuff!
- We will also recover some material in Schutz Chapter 4 on the stress-energy tensor, particularly section 4.7

Problems:

(1) A metric perturbation has the form

$$h_{\alpha\beta}(t,x) \to \begin{pmatrix} b & 0 & 0 & 0 \\ 0 & -b & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & -a \end{pmatrix} \sin(kx - \omega t)$$

with $\omega = k$ and constants a and b.

(a) Show that this is a solution to the linearized Einstein equations in vacuum, $\delta R_{\alpha\beta} = 0$.

(b) Recall from our discussion that even when $h_{\alpha\beta}$ satisfied the Einstein equations the transformation functions ζ_α had to satisfy $\Box\zeta_\alpha = 0$. Given this and the transformations

$$h_{\alpha\beta} \to h'_{\alpha\beta} = h_{\alpha\beta} - \partial_\alpha\zeta_\beta - \partial_\beta\zeta_\alpha$$

find the transforms that change the above form of $h_{\alpha\beta}$ to one where $b = 0$. Find the value of a after this transformation.

(c) Describe this final wave solution.

(2) 9.8 Does the soup slosh? Please be complete in your discussion. For example what if the soup bowl diameter is about the wavelength of the gravitational wave?

(3) 9.12 Filling in part of the calculation we left out in class. Please skip the part we didn't do, on equation 9.32.

(4) 9.26 (2 pts.) Detecting a wave