This first material is on special relativity with an emphasis on the foundations of Lorentz transformations and the geometry of flat spacetime. New notation, diagrams, methods of calculating, and concepts abound!

All numbered problems are from Schutz. The chapter is given first, then the problem or ‘exercise’ in Schutz’s lingo, e.g. 1.13 is the 13th exercise of Chapter 1.

**Reading:**

Chapter 1 in Schutz.

Other useful resources include Einstein’s 1905 SR paper (there’s a link on this page), Ellis and Williams, Mermin, and Rindler.

**Problems:** Due Thursday, Jan. 25 in class.

1. Work out the following in \( c = 1 \) units. (See 1.1 for a worked example.)
   a. 1 J (You can see why we might not use this “c=1” set of units in intro lab!)
   b. 1 eV (“electronvolt” - an energy often used in atomic and particle physics)
   c. 1 kWh
   d. 1 atm = \( 10^5 \) N m\(^{-2}\)

2. 1.3 “Locus” is defined by those points or events that satisfy the given condition

3. 1.10 ‘ID’ing intervals

4. (Postponed to next week) 1.12 This should help resolve confusion about reciprocity and time dilation: Yes, they both can view the other’s clock as running slow!

5. (Postponed to next week) 1.14 Taylor series! Can you explain why Lorentz and others did not see linear corrections, \( v/c \), but had to go to second order \( (v/c)^2 \)?

6. (Postponed to next week) Cosmic rays are particles that arrive at Earth traveling at extremely high relative velocities \( (v \sim 0.99) \). Their origin remains something of a mystery. At a height of about 20 km these particles collide with atoms in the upper atmosphere producing a shower of high energy particles including particles called muons. These particles decay rather quickly; as measured in the laboratory a muon at rest has a mean lifetime of \( 2.197 \times 10^{-6} \) s. (You may have observed this yourself!) Suppose that a muon is created at a height of 20 km moving at \( v = 0.99 \).
   a. Find the flight time to earth.
   b. Defining \( f := (\text{mean time of flight})/(\text{mean lifetime}) \), find \( f \) for such muons.
   c. The fraction of muons that survive is given by \( e^{-f} \). How large is this? It turns out that 1% survive. Does your number agree?
   d. If not, explain what went wrong with this calculation and correct it.

7. (Postponed to next week) A Federation cruiser is at rest relative to the border of Klingon Space and is in Federation Space. In the Federation cruiser’s frame the border is 6 light-hours away. A Klingon battleship zips by the cruiser moving at \( \frac{2}{3}c \) towards the border. Let’s call this event A and have it designate noon for both frames. A little while later the Klingon battleship fires a parting shot in the form of a laser beam (“phaser”) at the Federation cruiser. The phaser impacts the cruiser at 8 PM, according to the Federation clocks, severely damaging the spaceship.
(a) Sketch a space-time diagram of the history in the cruiser’s frame, including the spaceships, border, and phaser.
(b) According to the Federation cruiser’s clocks, when does the Klingon ship pass into Klingon Space? Explain how you found this result.
(c) According to the Federation clocks, when did the Klingon battleship fire the phaser? Explain.
(d) Describe the history of events in the Klingon’s frame.
(e) Many months later the case comes to Intergalactic Court. The Klingon-Federation Space-time Treaty states that it is illegal for a Klingon (Federation) ship in Federation (Klingon) territory to damage Federation (Klingon) property. The lawyer representing the Klingon ship’s captain argues that they are within the letter of the law, since in the ship’s frame the damage to the cruiser occurred after the Klingon ship crossed back into Klingon territory. Hence they were not in Federation territory at the time the damage occurred. Did the event of the phaser impacting the cruiser really occur after the Klingon ship crossed into Klingon territory, in their frame?
(f) On the basis of this case, would you recommend that the Intergalactic Congress re-negotiate the treaty to clarify this law? If so, how would you recommend wording the treaty? If not, what is your advice to ship captain’s?

(8) **Optional** Write your own rigorous derivation of the Lorentz transformations. Use any method but highlight necessary assumptions.

(9) **Not required!** (I’d love to hear your thoughts) In addition to the principle of relativity, suppose that there is an invariant energy scale $E_\ast$ and that the speed of light depends on the energy,

$$c(E) = c \left( 1 - \frac{E}{E_\ast} \right),$$

where the photon’s energy is $E < E_\ast$. (We think about this relation as giving first term in a Taylor series.) What are new the Lorentz transformations? Determine the physical consequences including time dilation and length contraction.