General Relativity (PHYS 325) PS 6

Spring 2024 v2.5

This week we have a little Christoffel computing, work with geodesics on a sphere, and more on accelerating observers. It looks like we are on track to assemble Einstein's equation before break. After break we will study the main current applications: Gravitational waves, black holes, and cosmology.

Reading:

- Schutz Chapter 6 sections 3 - 7

Problems:

All numbered problems are from Schutz.

(1) When we discussed the example of parallel transport on a sphere I just stated that great circles were geodesics.

(a) Show that the geodesic equations for a sphere of radius a are

$$\begin{aligned} \frac{dU^\theta}{d\lambda} &= \cos\theta\sin\theta\,(U^\varphi)^2 \\ \frac{dU^\varphi}{d\lambda} &= -2\cot\theta\,U^\varphi U^\theta \end{aligned} \tag{1}$$

(b) Noting that $U^i = dx^i/d\lambda$ express the geodesic equations in terms of the coordinates $x^i(\lambda)$.

(c) Show that on the equator, $\theta = \pi/2$ and $\varphi = a\lambda + b$ satisfy the geodesic equation. Explain how we can choose $\varphi = \lambda$, without loss of generality.

(d) Explain why this solution at the equator applies to every great circle.

(2) "Oops!" What is wrong with the following equation?

$$\frac{dp^\mu}{d\tau} = qF^{\mu\nu}U^\beta$$

Using the metric how might you correct the typo?

(3) Compute the Christoffel symbols Γ^t_{tt} and Γ^i_{tt} for the metric shown in equation (7.8) assuming that the function $\phi = \phi(t, x, y, z)$ is a function of all the coordinates.

(4) 5.21 The last of the "Rindler spacetime set" of 2.19, 2.21, and 5.21.