Introduction to Mathematica for GR

April 2020

Syntax

For the purposes of this story there are five things you need to know about mathematica:

1. The commands and functions start with Capital letters e.g. Sin
2. The arguments of functions are denoted with square brackets e.g. Sin[x] and inside these options have curly brackets
3. To ask mathematica to compute press **enter** or *shift* **return**
4. Help is found in Wolfram Documentation under the "Help" menu - it is excellent
5. It is in command line format so you can refer to earlier entries later.

(with apologies to Mo Willems.)

Scope

Incorporating symbolic manipulation, a coding environment, and numerical routines, software like mathematica is extremely powerful.

For example:

- You can compute and manipulate tensors!!
- You can plot
- You can integrate analytically and numerically
- You can plot data and combine plots using Show
- You define functions - use underscore to define dependent variables - differentiate, and manipulate functions
- You can solve differential equations analytically and numerically

Working with the software

- Plot $sin(x)$ from 0 to 3π with Plot[Sin[x],{x,0,3 Pi}]
- Define this plot to be g1 with g1 = Plot[Sin[x],{x,0,3Pi}] .
- Define the plot of $cos(x)$ from 0 to 3π to be g2. Now type Show[g1,g2] - it combines the plots!
- Go to the Help menu and select "Wolfram Documentation". In the new window search for integrate. Working from the examples at the bottom of the information page integrate

$$\frac{x^3}{(e^x - 1)} \text{ from 0 to } \infty.$$

- In the Insert menu find "Inline free form..." and select it. Now you have Wolfram Alpha. Try typing in english "plot sin(x) ..."
- See what else you can do with alpha.

GR I: Tensors

- Download the robertson-walker k=0 mathematica notebook, text file, or html from the course website
- Try running the lines to the matrix form. Verify that the metric looks correct in matrix form.
- If so evaluate the rest by evaluating the whole notebook or pressing enter for the remaining lines. Verify that G_{tt} and G_{rr} are correct.

Solving diff equ'ns I

Mathematica has a differential equation solver allowing us to solve differential equations analytically and numerically.

- For analytic solutions we can use DSolve. Try solving this differential equation from a recent problem set

$$\frac{dT}{dt} = -\alpha T + \beta$$

  by entering
  DSolve[y'[t] == - α*y[t] + β,y[t],t]
  Note the notation for derivative and how we ask for the solution $y(t)$ in terms of t.
- I have found DSolve (and the numerical version NDSolve) to be the most persnickety of mathematica commands. Work from examples and be patient.

GR II - general metrics

- Download the einstein notebook (or text file)
- Enter in the components for 'our metric' with spherical symmetry. Don't forget to enter $\Phi[r]$ so mathematica knows this is a function of the r coordinate. Verify that it looks correct in matrix form.
- Ask it to compute the Christoffels. Are they correct?
- If so evaluate the rest by evaluating the whole notebook or pressing enter for the remaining lines. Verify that G_{tt} and G_{rr} are correct.