Stat Mech (PHYS 370) Guide 11

Fall 2024 v1.0

In the week after Thanksgiving break we will finish our discussion of the Sommerfeld expansion before deriving the blackbody spectrum.

Normally we finish out the semester with topics such as phase transitions, which were first discussed in Chapter 5, heat engines, phonons in solids, and Bose-Einstein condensation. However, I am tempted for open up his up to some alternative projects. I'll mention these in class Thursday Nov 21 so we have some time to mull over the possibilities before deciding our next adventure in statistical physics.

Reading:

Chapter 7 sections 7.3 and 7.4

Problems: Due before class on Thursday Dec. 5

(1) Andrew Projanski, a former Hamilton physics student, proposed the following formula for boson partition function

$$Z_b = \frac{Z_1^N - Z_1}{N!} + Z_1.$$

- (a) Check that the formula holds for the 2 bosons in 4 states example we did in class. (We assumed all the energies were 0 so we could focus on the counting.)
- (b) Does this formula work for N bosonic harmonic oscillators?
- (c) Extra (optional) Prove the formula if it is correct or give a counter example if it is not.

(2) When are the three distributions close? Feel free to use Mathematica for this one.

- (a) At room temperature find how large $\epsilon - \mu$ must be for the Boltzmann, Bose-Einstein, and Fermi-Dirac distributions agree within 1%?
- (b) Consider an average molecule of our atmosphere with mass $\bar{m}_{\text{air}} \simeq 28.96$ g/mol from our solution to 1.14 and with the same rotational partition function as N_2 (see pg 236). By finding the value of μ on the surface (at 1 atm and 300 K) show that the limit you found in part (a) is not exceeded.
- (c) The coldest temperature of our atmosphere is about 100 K. Do we have to worry about quantum counting for our atmosphere at these temperatures? Comment on this more broadly in terms of planetary atmospheres.

(3) 7.23 (4 pts.) Writing up the full results for the white dwarf stars and the Chandrasekhar mass limit. In part (a) obtain the constant, as we did in class.

(4) Use Mathematica to evaluate the integrals

$$\int_{-\infty}^{\infty} \frac{x^n e^x}{(e^x + 1)^2} dx$$

for $n = 0, 1, 2,$ and 3.