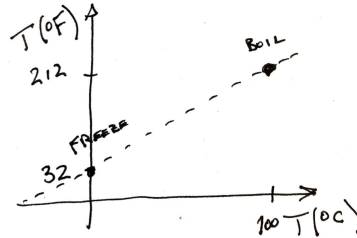


Solutions:

(1) Here's a sketch of the relation:



It is clear that we can obtain a linear relation between temperature in Fahrenheit, $T(^{\circ}F)$, from temperature in Celsius $T(^{\circ}C)$. Reading from the sketch,

$$T(^{\circ}F) = \alpha T(^{\circ}C) + b$$

for some slope α and intercept b . From the sketch, $b = 32$.

To find α , let's use the boiling point of water at $100^{\circ}C$ and $212^{\circ}F$,

$$212 = \alpha(100) + 32 \implies \alpha = \frac{212 - 32}{100} = 1.8 = \frac{9}{5}.$$

so

$$T(^{\circ}F) = \frac{9}{5}T(^{\circ}C) + 32.$$

(For fun you could re-run the same argument to obtain

$$T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) - 32]$$

Or you could just invert the first version of course.)

Nice to recover these familiar relations!

(2) Temps in kelvin

(a) I recall $98.6^{\circ}F$ as healthy at rest. Using the above formula and adding 273 gives $\frac{5}{9}(98.6 - 32) + 273 = 310$ K.

(b) 373 K

(c) The hottest I recall is $110^{\circ}F$ in Phoenix. This is $\frac{5}{9}(110 - 32) + 273 \simeq 416$ K.

(d) Adding another sig fig, $273.15 - 268.9 \simeq 4.2$ K

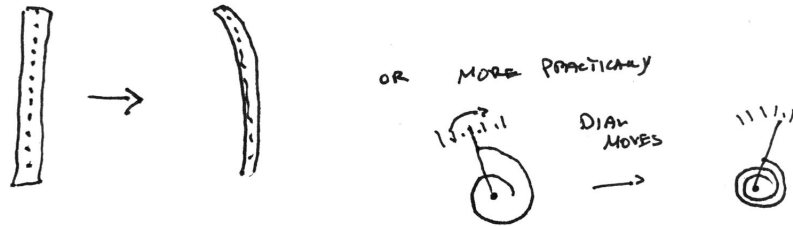
(3) In recent years this has been a IR thermometer for me. In principle this is instantaneous due to the radiation spectrum of objects at any (non-vanishing) temperature. (We'll see why in Chapter 7!) Long ago when I used mercury thermometers, the glass and mercury had to reach equilibrium with my mouth. Seemed to take forever but the relaxation time was probably only a couple of minutes.

- (4) Two objects at the same temperature, such as a metal slide and powdery snow on a cold winter morning, feel quite different. Our nervous system is set to record the heat Q not the temperature T . This is a better idea since excessive heat flow can harm us unless we break contact. Both heat capacity, conductivity, and quality of contact help determine how “hot” or “cold” an object feels.
- (5) Thermal expansion
- (a) For a 1 km steel bridge the difference in length between a cold winter day at $-20^\circ\text{F} \simeq -29^\circ\text{C}$ and a hot summer day at $100^\circ\text{F} \simeq 38^\circ\text{C}$ would be

$$\Delta L = \alpha L \Delta T \simeq 73 \text{ cm.}$$

I'd leave a bit more than that if possible - 80 cm? The errors mentioned in the problem mean that the engineers were designing the rails and airports for a more limited temperature range - and probably did not round up like I just did. Of course, steel bridges are not built from 1 km steel beams. Instead they are much shorter, with expansion joints built in, which we often feel as a bump as we drive over them.

- (b) The differing thermal expansion coefficients cause the bimetallic strips to curve somewhat like this



- (6) A mole has $\sim 10^{23}$ particles and has a mass of about a gram. The density of air is 1.23 kg/m^3 . In a $10 \text{ m} \times 10 \text{ m} \times 1 \text{ m} = 10^2 \text{ m}^3$ classroom there is about

$$m \simeq 1.23 \cdot 10^2 \simeq 100 \text{ kg}$$

of air. Thus,

$$N \simeq 123 \cdot 10^3 \cdot 10^{23} \simeq 10^{28}.$$

Alternatively,

$$N \frac{PV}{kT} = \frac{(10^5 \text{ Pa})(10^2 \text{ m}^2)}{1.38 \times 10^{-23} \text{ J/K}(293 \text{ K})} \simeq 10^{27},$$

roughly the same.

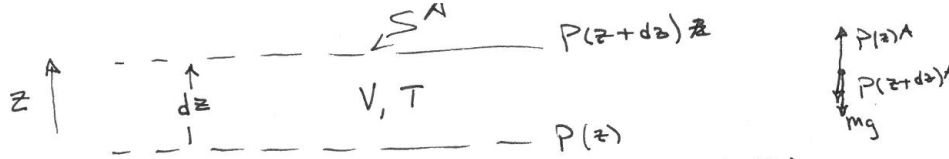
- (7) The mass of moles
- (a) Water is H_2O so that's about $2+16 = 18$ nucleons so a mole is about 18 g. The numbers are from the periodic table.
- (b) As for N_2 , it is $2 \times 14 = 28$ nucleons so a mole is about 28 g.
- (c) Lead is Pb with 207 nucleons so one mole is 207 g.
- (d) SiO_2 has $28 + 2 \times 16 = 60$ nucleons so one mole is 60 g.

- (8) The average molar mass of air (by volume) is the the sum of the contributions of nitrogen (28 g for 1 mole) , oxygen (32 g), and argon (40 g) (all molar masses)

$$\bar{m}_{\text{air}} = (0.78)(28 \text{ g}) + (0.21)(32 \text{ g}) + (0.01)(40 \text{ g}) \simeq 28.96 \text{ g/mol} \simeq 2.9 \times 10^{-2} \text{ kg/mol}$$

BTW, where's CO₂ is ~400 parts per million or 0.04%.

- (9) We started this one in class. Let's consider a slab of air between heights z and $z + dz$:



The sketch sets some of the notation and the FBD gives

$$P(z)A - P(z + dz)A - mg = 0.$$

For a stationary slab of air with horizontal area A . Using $m = \rho V$ and doing a Taylor expansion on $P(z + dz)$ gives

$$P(z)A - P(z)A - \frac{\partial P}{\partial z} dz - \rho A dz g = 0 \text{ or } \frac{\partial P}{\partial z} = -\rho g,$$

after cancelation. The final equation looks nice and simple but not what we want since the differential equation has pressure on one side and density on the other. This can be fixed since

$$\rho = \frac{m}{V} = \frac{\bar{m}_{\text{air}} N}{V} = \frac{\bar{m}_{\text{air}} P}{kT},$$

using the ideal gas law. Thus,

$$\frac{\partial P}{\partial z} = -\frac{\bar{m}_{\text{air}} g}{kT} P$$

This is great: P's on both sides and since we are assuming T is constant, the factor of the right hand side is just a constant. So integrating gives

$$\int_{P(0)}^{P(z)} \frac{dP}{P} = -\frac{\bar{m}_{\text{air}} g}{kT} \int_0^z dz \implies \ln \left(\frac{P(z)}{P(0)} \right) = -\frac{\bar{m}_{\text{air}} g}{kT} z.$$

Or

$$P(z) = P(0) e^{-\frac{\bar{m}_{\text{air}} g}{kT} z}$$

as desired. For conditions, $P(0) = 1 \text{ atm}$ and $T = 285 \text{ K}$ at sea level, the constant is

$$\frac{\bar{m}_{\text{air}} g}{RT} \simeq 1.2 \times 10^{-4} \text{ m}^{-1} \simeq (8300 \text{ m})^{-1}.$$

Density follows the same exponential fall-off since, as I noted above,

$$\rho = \frac{\bar{m}_{\text{air}}}{kT} P,$$

so the density and pressure are directly proportional and so

$$\rho(z) = \rho(0) e^{-\frac{\bar{m}_{\text{air}} g}{kT} z}.$$

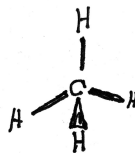
Using this model the given sites have pressures of

$$\text{Hamilton: } P \simeq 0.97 \text{ atm} \simeq 9.7 \times 10^4 \text{ Pa}$$

$$\text{Mt. Marcy: } P \simeq 0.82 \text{ atm} \simeq 8.2 \times 10^4 \text{ Pa}$$

$$\text{Schilthorn: } P \simeq 0.70 \text{ atm} \simeq 7 \times 10^4 \text{ Pa}$$

- (10) It is helpful to sketch the chemical form of this. As described in the problem the four hydrogens sit on the vertices of a tetrahedron and the carbon sits in the middle. Here's a sketch:



It all looks fairly floppy so here's what I would expect in terms of degrees of freedom: translation contributes 3 and rotation of the whole thing contributes 3. Schroeder tells us that each vibrational mode contributes 2 degrees of freedom. So vibrationally the degrees of freedom have $2 \times 4 = 8$ -ish for each of those bonds if there is enough energy to excite these degrees of freedom so $f = 3 + 3 + 8$ would be 14.

Alternatively, if the molecule looks completely floppy then each atom can move independently and we could just say that it has degrees of freedom equal to $f = 3N = 15$ for $N = 5$ atoms.

The actual numerical answer is not important at this point, the reasoning is. Of course there is a correct answer. It is 15.