

Solutions:

- (1) To find the rms speed of the nasty sounding gas we can use the molar masses M of the gases of the two isotopes;

$$v_{rms} = \sqrt{\frac{3kT}{m}} \sqrt{\frac{3RT}{M}}.$$

For U-235,

$$M_{235} \simeq 235 + 6 \cdot 19 \simeq 349\text{g}, \text{ and for U-238 } M_{238} = 352 \text{ g}.$$

So the speed at room temperature is about $v_{rms} \simeq 146 \text{ m/s}$. These masses are not all that different. The ratio of speeds is

$$\sqrt{\frac{352}{349}} \simeq 1.0043$$

Yikes! This is close. But, surprisingly, it is a frequently used technique!

To separate these isotopes using a centrifuge we could fill it with UF_6 gas (containing a mixture of U-235 and U-238). Once the gas is spinning (fast!) we could draw out the gas from the center of the centrifuge. This gas will be the less-dense UF_6 with a (slightly) higher percentage of U-235. We could then send this gas onto another centrifuge, and another, and another ... until the gas is more refined. Clearly, this is a large industrial operation.

- (2) For an “ideal diatomic” molecule there are 3 translational degrees of freedom and 2 rotational degrees of freedom (recall the quantum reduction), giving $f = 5$. (If you chose a classical model with degrees of freedom of $f = 3 + 3 = 6$ for translations and rotations, no worries but now you know for the future!)

- (a) For **process A** there is no change in volume so there is no work done. Hence by the first law, $\Delta U = Q$. By the ideal gas law we know that

$$P_1 V_1 = NkT_1 \text{ and } P_2 V_1 = NkT_2.$$

Solving for the change in temperature

$$\Delta T_A = T_2 - T_1 = \left(\frac{V_1}{Nk} \right) (P_2 - P_1).$$

Now, from the equipartition theorem the change in energy for this process is

$$\Delta U_A = \frac{5}{2} Nk \Delta T_A = \frac{5}{2} V_1 (P_2 - P_1).$$

Since $W_A = 0$ we have

$$\Delta U_A = Q_A = \frac{5}{2} V_1 (P_2 - P_1).$$

In summary,

$$\text{For process A } \Delta U_A = \frac{5}{2} V_1 (P_2 - P_1), Q_A = \frac{5}{2} V_1 (P_2 - P_1), \text{ and } W_A = 0.$$

For **process B** the pressure stays the same but volume changes so we have some work as well as Q . The work is

$$W_B = -P \Delta V = -P_2 (V_2 - V_1).$$

I'll call the temperature on the upper right to be T_3 . So $P_2V_2 = NkT_3$. Then

$$\Delta U = \frac{5}{2}kN(T_3 - T_2) = \frac{5}{2}P_2(V_2 - V_1).$$

Hence,

$$Q = \Delta U - W = \frac{7}{2}P_2(V_2 - V_1).$$

In summary,

For B $\Delta U_B = \frac{5}{2}P_2(V_2 - V_1)$, $Q_B = \frac{7}{2}P_2(V_2 - V_1)$, and $W_B = -P_2(V_2 - V_1)$.

The others processes are similar giving,

$$\text{for C } \Delta U_C = -\frac{5}{2}P_2(V_2 - V_1), Q_C = -\frac{5}{2}P_2(V_2 - V_1), \text{ and } W_C = 0.$$

and

$$\text{for D } \Delta U_D = -\frac{5}{2}P_1(V_2 - V_1), Q_D = -\frac{7}{2}P_1(V_2 - V_1), \text{ and } W_D = P_1(V_2 - V_1).$$

(If you had $f = 6$ then the numerical prefactors are different, e.g. $5/2 \rightarrow 3$.)

- (b) Process A is a no-work process in which the pressure increases. Then by the ideal gas law the temperature increases; heat must flow in, say by heating it with a burner. Process B is an expansion at constant pressure; a piston would move out. Work is done. The temperature and energy increase. So heat flows into the system to maintain a constant pressure. For process C, only the pressure changes so no work is done. Pressure decreases and so does the temperature. The energy decreases so heat flows out of the system, perhaps by holding it against a block of ice. Process D is a constant pressure process again. The system contracts; work is done. Heat flows out again and temperature drops to the original value.
- (c) The net quantities are obtained by adding them up.

$$\Delta U = 0, Q = (P_2 - P_1)(V_2 - V_1), \text{ and } W = -(P_2 - P_1)(V_2 - V_1).$$

Does this make sense? Well, after one cycle we are back to the same state as before so the change in energy must vanish and it does! The other two must sum to zero, and they do. Finally, the net work is *negative* so the system of gas does work on its surroundings. This sign makes sense since the pressure when the gas is expanding (P_2) is larger than the pressure when the gas is contracting (P_1). The energy to support this work comes from the heat and so $W = -Q$ as expected. Heat is converted to work in this cyclic process.

- (3) The rising bubbles enjoy different processes: Bubble A rises so that no heat flows ($Q = 0$) - "A for adiabatic". Bubble B rises so that the temperature inside does not change; it is an isothermal process.

Let's assume the ideal gas law for both bubbles,

$$PV = NkT \text{ or } V = \frac{Nk}{P}T.$$

Since the bubbles are identical N and the *initial* T are the same. As they rise $P = P(z)$ is also the same. Hence, the volume is controlled only by the temperature. For the adiabatic bubble A, $Q = 0$ and

$$W = - \int PdV < 0 \text{ due to the expansion.}$$

So since $\Delta U = W$ and $U \propto T$ then the energy and temperature in bubble A must fall.

For bubble B the temperature is constant so the change in volume is controlled only by the behavior of P only. Bubble B then has a higher temperature and a higher volume (the pressures in the two bubbles are the same at any given depth).

Alternatively, you can reach the same conclusion using the results from class:

$PV = \text{const}$ for isothermal processes, and $PV^\gamma = \text{const.}$ for adiabatic processes

so that

$$\Delta V = -\frac{V \Delta P}{\gamma P}$$

for adiabatic processes and

$$\Delta V = -\frac{V \Delta P}{P}$$

for isothermal processes. Since $\gamma > 1$ the change in volume for bubble A will be smaller.

- (4) Decreasing T and convection in the atmosphere. You could start off with $\Delta U = W = -P\Delta V$ and proceed from there but I will draw on our work in class.

(a) As we saw just before I defined γ ,

$$VT^{f/2} = \text{const.}$$

for an adiabatic process. With the ideal gas law we can re-express V in terms of T and P in this expression, giving

$$\frac{1}{P}T^{f/2+1} = \text{const.}$$

Differentiating this gives

$$-\frac{dP}{P^2}T^{f/2+1} + \left(\frac{f}{2} + 1\right) \frac{1}{P}T^{f/2}dT = 0$$

Or,

$$\frac{dT}{dP} = \frac{2}{f+2} \frac{T}{P}$$

as desired.

- (b) From the above result for dT/dz

$$dT = \frac{2}{f+2} \frac{T}{P} dP$$

which is the change in temperature under an adiabatic change to the pressure. If this change occurs over a vertical distance dz and if forces are balanced - setting the critical value for the onset of convection - then we can use the result from 1.16

$$dT = \frac{2}{f+2} \frac{T}{P} \frac{dP}{dz} dz$$

or

$$\frac{dT}{dz} = -\frac{2}{f+2} \frac{\cancel{T} \bar{m}_{\text{air}} g}{\cancel{P} k \cancel{T}} = -2 \frac{\bar{m}_{\text{air}} g}{k(f+2)} = -2 \frac{\bar{M}_{\text{air}} g}{R(f+2)} \simeq 9.8 \text{ K/km.}$$

I have used $f = 5$ and $\bar{M}_{\text{air}} = 29 \text{ g/mol}$. This “dry adiabatic lapse rate” works out to be about $5.4^\circ\text{F} / 1000 \text{ ft}$.

- (5) Just like what we might do in an intro physics lab. Heat flows from the metal to the water in the styrofoam cup.

- (a) Since water has $c = 4.2 \text{ J/}^\circ\text{C}$ we have

$$Q = mc\Delta T = (250 \text{ g})(4.2 \text{ J/}^\circ\text{Cg})(4^\circ\text{C}) = 4.2 \text{ kJ.}$$

- (b) We assume that the system is isolated so all this heat flows from the metal, $Q = 4.2 \text{ kJ}$.
 (c) Since the metal went from 100°C to 24°C then

$$C = \frac{Q}{\Delta T} = \frac{4.2 \text{ kJ}}{76^\circ\text{C}} \simeq 55 \text{ J/}^\circ\text{C}$$

- (d) Dividing by the mass, the specific heat is $c \simeq 0.55 \text{ J/ g }^\circ\text{C}$.

(6) Partial derivatives

- (a) Since $y = x/z$, $w = xy = x^2/z$. Also, $w = y^2z$.
 (b) Now using $w = xy$

$$\left(\frac{\partial w}{\partial x}\right)_y = y = \frac{x}{z} \text{ while using } w = x^2/z \text{ the partial derivative } \left(\frac{\partial w}{\partial x}\right)_z = \frac{2x}{z}$$

So these are not equal.

- (c) Completing the set:

$$\left(\frac{\partial w}{\partial y}\right)_x = x = yz \text{ and } \left(\frac{\partial w}{\partial y}\right)_z = 2yz$$

and

$$\left(\frac{\partial w}{\partial z}\right)_x = -\frac{x^2}{z^2} \text{ and } \left(\frac{\partial w}{\partial z}\right)_y = y^2 = \frac{x^2}{z^2}.$$

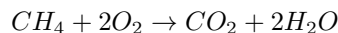
So again these are not equivalent.

- (7) A spring snow melt Fermi problem. Your estimates are likely to be a bit different than mine but we should have results at the same order of magnitude. Schoeder sets this up so that we looking at melting a 1 m^3 block of ice. This has a mass of about 917 kg . Since the latent heat of ice is 333 J/g this requires about $3 \times 10^8 \text{ J}$ to melt. Let's assume that the air temperature is just above freezing. If the sun shines about 8 hours a day then we have

$$0.1 \cdot \frac{10^3 \text{ J}}{\text{s}} \cdot \frac{3600 \text{ s}}{\text{hour}} \cdot 8 \text{ hours} \simeq 2.9 \times 10^6 \text{ J}$$

of energy heating the 1 m^2 face of the block per day. I've taken 10% since the problem states that 90% is reflected. Dividing these two one gets about 100 days or about 15 weeks.

- (8) Combustion and enthalpies all for the burning of natural gas,



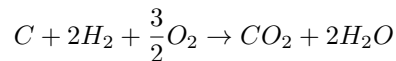
- (a) The book gives enthalpies of formation on pages 404-5, that is



Breaking up the methane then gives $= 74.81 \text{ kJ}$.

- (b) The change in enthalpy to form one mole of CO_2 is -393.51 kJ . Similarly the change in enthalpy to form *two* moles of H_2O is -483.64 kJ . (The total is about -877 kJ .)

- (c) We can compute the enthalpy in two steps, first break up the reactants, $CH_4 \rightarrow C + 2H_2$, and then form the products



so that

$$\Delta H = 74.81 - 393.51 - 483.64 \simeq -802 \text{ kJ}$$

per mole of methane. Even though the reaction doesn't follow these steps, since the products and reactants are the same, the net change in enthalpy is the same.

- (d) In a constant-pressure reaction when no other work is done, the decrease in enthalpy is all heat; $Q = 802 \text{ kJ}$.
- (e) In this constant pressure process the reaction starts with three moles of gas and ends with three moles of gas. So there is essentially no change in volume. With no change in volume the change in energy is the change in enthalpy, $\Delta U = -802 \text{ kJ}$.

On the other hand if the water ends in a liquid state then the situation changes. The enthalpy of formation of (two moles of) liquid water is now -571.66 kJ , more than before. The net change in enthalpy is now $\Delta H = -890 \text{ kJ}$. But in addition we end up with only one mole of gas. From the ideal gas law

$$\Delta V = \frac{RT}{P} \Delta n = -\frac{2RT}{P}$$

and

$$\Delta U = \Delta H - P\Delta V = \Delta H + 2RT = -890 + 5 \simeq -885 \text{ kJ}$$

at 298 K. This means that the heat given off is a bit larger than the energy lost by the system. This is due to the compression of the system by the atmosphere.

- (f) To produce this reaction we need a mole of methane (16 g) and two moles of oxygen (64 g) for a total of 80 g. If the sun were comprised of these molecules in this ratio then it would have $2 \times 10^{30}/0.08 = 2.5 \times 10^{31}$ moles of methane. This would produce about $2.5 \times 10^{31} \times 8 \times 10^5 \simeq 2 \times 10^{37} \text{ J}$ of heat. At a rate of $4 \times 10^{26} \text{ W}$, the sun would only last $5 \times 10^{10} \simeq 1600 \text{ yrs!}$ Methane combustion is certainly not the reaction that fuels the sun!
- (g) For every mole of methane we obtain a mole of CO_2 . We need the volume of a mole

$$V = \frac{RT}{P} \simeq 0.025 \text{ m}^3 \simeq 0.88 \text{ ft}^3$$

under standard conditions. We also need the molar mass of CO_2 - it is 12 g for C and 32 g for O_2 for a total of 44 g. Hence,

$$m_{CO_2} = 24 \times 10^6 \text{ ft}^3 \cdot \frac{1 \text{ mol } CO_2}{0.88 \text{ ft}^3} \cdot \frac{44 \text{ g}}{1 \text{ mol } CO_2} \simeq 1.2 \times 10^6 \text{ kg} = 1.2 \text{ Mton } CO_2$$

- (9) A start on modeling heat flow in homes, here just conduction through still air. For 3.5" of still air with thermal conductivity 0.026 W/mK we have

$$R = \frac{\Delta x}{k_t} \simeq 3.4 \text{ m}^2 \text{ K/W}$$

The units in the US are 5.67 times larger so this air insulation would have an R-value of 19. (If you'd like to know the US unit is $^\circ\text{F ft}^2 \text{ hr/Btu}$.) This is much bigger than the R-values mentioned in the text. The difference is due to convection: the batting in the insulation prevents some convection but not all.

- (10) Using the Fourier heat conduction law we can compute the energy loss through walls, roof (or ceiling), windows, and floor.

- (a) The heat flow or loss rates are

$$\frac{dQ}{dt} = -\frac{A}{R}\Delta T$$

for areas A , R-value R , and temperature difference ΔT . I assumed that the house had a cathedral ceiling and the roof was insulated. (This is different than a traditional house in which the attic would be basically at the outside temperature and the floor of the attic would be insulated.) The heat loss was therefore through the walls, windows, roof, and floor. I computed “loss factors” = A/R for these channels in column G. The rates of energy loss are in column H. The total, summed at the bottom of column H, is about 11.5 kW. (If you assumed a traditional home with ceiling insulation, this would be a little lower, about 11.3 kW.)

- (b) This is simply the whole house loss rate (11.5 kW) times the number of hours in a day. I found a total of about 280 kWh.
- (c) I used the energy prices (and the conversion of 29 kWh/therm) to compute the costs: \$68 for electricity¹ and \$17 for gas. It turns out that natural gas is cheaper. This is one reason why many homes are heated with natural gas.

Here’s my spreadsheet:

¹This is for an electric resistive heat not a heat pump. We’ll come back to heat pumps later. They totally change the picture.

Box house energy computations

Box house geometry

width (m) 8 depth (m) 12 height (m) 1.5 roof height from ceiling (m) 4

Windows width (m) 0.75 height (m) 1.5
 R-value 0.04 R-value (US) 0.2268 Loss factors (A/R) 225 Energy Flux (W) 6250 Daily heat loss (J) 5,400E+08 Daily heat loss (kWh) 1.500E+02

Number 8 window area (m²) 9

Walls (m²) 172 R-value 3 R-value (US) 17.01
 163 without windows

Roof (m²) 136 R-value 5 R-value (US) 28.35

Volume 528 m³

Floor area 96 m² R-value 0.17636684 R-value (US) 1

Constant ground temperature (deg C) 12.777778

Inside T 65 outside T 15 Delta T 15 hours 27.777778
 18.3333333 -9.4444444 27.777778
 291.3333333 263.5555556

Energy Prices 0.245 \$/kWh
 2.4 Electricity unit convert J to kWh 29.3 2.7778E-07

Nat. Gas 1.8 \$/therm
 Daily Costs electric resistance heater (100% efficient) \$67.85
 natural gas furnace (100% efficient) \$17.01

a 11539 b 7.258E+01
 2.769E+02