Solutions:

(1) If $T_A = T_B$ and $T_B = T_C$ then

$$\frac{\partial S_A}{\partial U_A} = \frac{\partial S_B}{\partial U_B} \text{ and } \frac{\partial S_C}{\partial U_C} = \frac{\partial S_B}{\partial U_B}$$

Then

$$\frac{\partial S_A}{\partial U_A} = \frac{\partial S_C}{\partial U_C}$$

by transitivity. Hence $T_A = T_C$.

(2) From (2.17) the multiplicity in the low temperature limit $q \ll N$

$$\Omega \simeq \left(\frac{eN}{q}\right)^q.$$

Since $U = q\epsilon$ we have

$$S \simeq k\frac{U}{\epsilon}\left[\ln\left(\frac{N\epsilon}{U}\right) + 1\right]$$

Taking the partial derivative with respect to U gives us (after some algebra)

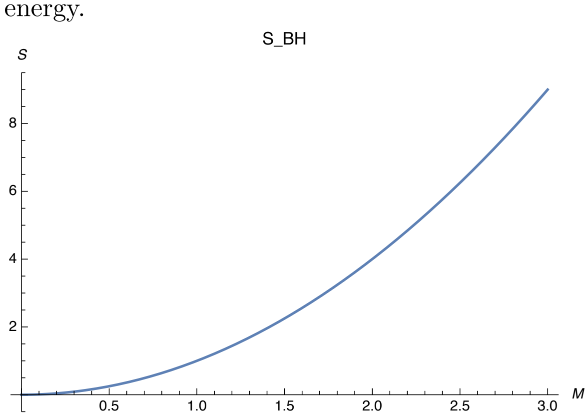
$$U = N\epsilon e^{-\epsilon/kT}$$

As T goes to zero so does the energy.

(3) Using our expression from before and the fact that $U = Mc^2$ we differentiate to find

$$T = \frac{hc^3}{16\pi^2 Gk\, M}$$

the celebrated Hawking temperature. For a solar mass black hole this works out to be about 6×10^{-8} K. That is cold! The sketch of the entropy is a curve starting at zero and rising as a quadratic function of energy.



So as energy (mass) is added to the BH, the slope increases leading to a *decreasing* temperature. This is unusual since we usually expect that as energy increases, temperature does as well.

(4) Taking the derivative from the expression for U,

$$C = \frac{\partial U}{\partial T} = \frac{N\epsilon^2}{kT^2} e^{-\epsilon/kT}$$

This one is a good one for a plot. To see the full behavior it is useful to define a dimensionless temperature $t = kT/\epsilon$. C/kN is now the function $(e^{-1/t})/t^2$.

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In[1]:= Plot[(1/t^2)*Exp[-1/t], {t, 0, 4}]
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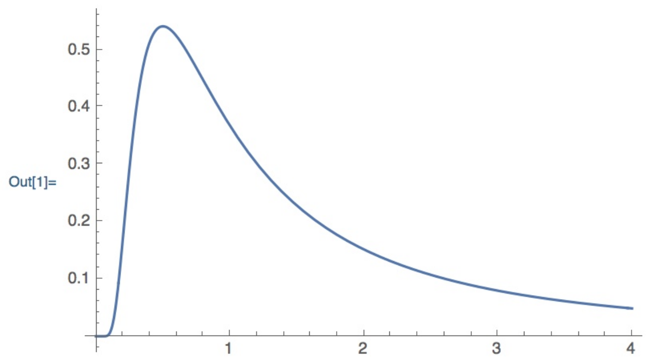


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In[2]:= Plot[(1/t^2)*Exp[-1/t], {t, 0, .5}]
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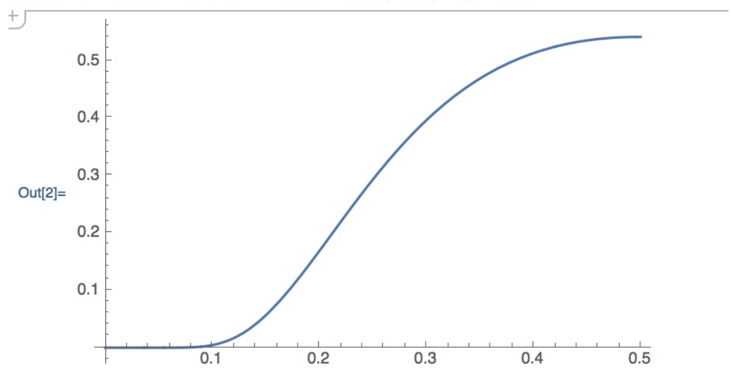

I have plotted C/kN on the full range to high temperatures, about 4 times the fundamental energy scale in the problem, and at low temperatures below this energy scale. You can easily see from the second plot that $C \to 0$ as $T \to 0$.

(5) Postponed

(6) Postponed

(7) Postponed

(8) Postponed

(9) Equilibrium temperature for Mars.

(a) The insolation, or sun's intensity, scales as r^2 so

$$I_{mars} = 1370 \cdot \frac{r^2_{earth}}{r^2_{mars}} \simeq 580 \text{ W/m}^2$$

(b) Without much of an atmosphere we have

$$\bar{P}_{sun} = \bar{P}_m ars \implies \mathcal{R} \cdot 580 \cdot \pi R_{mars}^2 = \sigma \cdot 4\pi R_{mars}^2 T_{equi}^4$$

or, including the reflectivity of $\mathcal{R} = 0.25$,

$$T_{equi} = \sqrt[4]{\frac{0.75 \cdot 580}{4\sigma}} \simeq 209.5 \simeq 210 \text{ K}.$$

The agreement is excellent. I guess there is essentially no greenhouse effect on Mars. This would be a tough place for Earth based life.

(10) Finding the energy released in gasoline combustion

(a) Using the table at the back of the text we find that for one mole of $H_2O(l)$ ΔH is -285.83 kJ. So for 9 moles we have $\Delta H \simeq -2572$ kJ. Likewise for CO_2, ΔH is -393.5 kJ. So for 8 moles we have the enthalpy of formation is $\Delta H \simeq -3148$ kJ.

(b) The octane number is given, oxygen doesn't contribute anything so

$$\Delta H = 249.7 - 3148 - 2572 = -5471 \text{ kJ}$$

This is negative so heat flows out of the system.

(c) From the definition of enthalpy we have $\Delta U = \Delta H - P\Delta V$ at constant pressure. So we need to find the work. With liquid water we see that we start with 12.5 moles of gas (oxygen) and end up with 8 moles of gas (CO_2) giving a change of $\Delta n = -4.5$ moles. The work done is then

$$-P\Delta V = -P\frac{\Delta nRT}{P} = -\Delta nRT \simeq 11 \text{ kJ},$$

using the change in the ideal gas law in the form $\Delta V = \Delta nRT/P$. The result uses the given temperature and is for one mole of gasoline. Thus, $\Delta U \simeq 5471 - 11 = 5460$ kJ/mol.

(d) The molar mass of C_8H_{18} is $8 \cdot 12 + 18 = 114$ g/mol. So dividing the energy by this gives -48 kJ/g gasoline. Finally, converting kJ to kcal (a factor of 4.184) gives the expected 11.4 kcal released by the (complete) combustion of 1 gram of gasoline.