Stat Mech (PHYS 370) Guide 8

Fall 2024 v1.0

We will finish up deriving the ideal gas law on Thursday before turning to more examples. We may discuss a couple more Chapter 6 topics like the Maxwell speed distribution before returning to counting and the statistical mechanics of quantum particles. We'll divide our time between bosons and fermions. Applications include degenerate fermi gases, blackbody radiation (including the cmbr), white dwarf stars, electrons in metals, and possibly Bose-Einstein condensation.

Reading:

Chapter 6 sections 2 - 7 (a good bunch of which we have discussed in class already)
Chapter 7 sections 1 - 2

Problems: (At the beginning of class Thursday November 7)

(1) 6.10 Exploring the vibrational energy levels of water

(2) 6.12 Interstellar gas clouds and, "Hello CMBR!"

(3) 6.13 A model of proton-genesis in the early universe.

(4) 6.16 An important result we use lots.

(5) 6.17 Fluctuations in energy in terms of derivatives of $\ln \mathcal{Z}$

(6) **Black hole partition functions I** Observers who are constantly accelerating not far outside the horizon of a black hole model the black hole spacetime as system of non-interacting atoms of geometry each with energy

$$E(j) = 2\epsilon\sqrt{j(j+1)}$$

where $j = 1/2, 1, 3/2, \ldots$ and

$$\epsilon = \frac{g\hbar}{2c},$$

where g is the observers' acceleration. (The reason for the funny factor of two will be come clear in a moment.)

(a) Show that for 'large' j the energy is given by

$$E(j) \simeq \epsilon\,(2j+1)\,.$$

(b) The degeneracy for the atoms of geometry is $d_j = 2j + 1$ for the degenerate states with different m_j values. Show that the one particle partition function can be written as

$$Z_1 = \sum_{m=2}^{\infty} m e^{-\beta\epsilon m}$$

where $m = 2j + 1$.

(c) Show that the partition function may be written as

$$Z_1 = x\frac{d}{dx}\sum_{m=2}^{\infty} x^m$$

where $x = e^{-\beta\epsilon}$. Using these derive this expression for the partition function in closed form

$$Z_1 = \frac{2 - e^{-\beta\epsilon}}{\left(e^{\beta\epsilon} - 1\right)^2}.$$

(d) Using Mathematica compare the large-j approximation to the exact result by plotting the two partition functions over the range of kT/ϵ from 0 to 1. You will have to truncate the exact result. Just try something not too big then check that your plot is insensitive to this maximum. Is there a domain of dimensionless temperature where the result agree? If so, what temperature limit does this correspond to?

(e) If there are a large number N of identical, indistinguishable geometric atoms, write down the partition function Z_N for N atoms in the large-j approximation.

(7) 6.44 a calculation of chemical potential

(8) 6.52 A relativistic gas - read 6.51 for an introduction to the approach.