Stat Mech (PHYS 370) Guide 9

Fall 2024 v1.5

We now study a series of systems with our new tools based on the partition function. Some of these are fun model systems, some important physical models with broad applicability and cool effects. The next big thing after our initial work counting quantumly will be particle statistics and fermionic systems, discussed at the beginning of Chapter 7.

Reading:
Chapter 7 sections 7.2 - 7.3

Problems: (Due on Thursday November 14 at the beginning of class)

(1) 6.20 The harmonic oscillator partition function! Ask me if your algebra long division is feeling rusty.

(2) 6.21 Modifying the simple harmonic oscillator energy. Use Mathematica (and our Mathematica intro... it will be very useful)

(3) 6.24 The partition function of oxygen

(4) 6.26 Heat capacity and low temp for the rotator system - see our class discussion

(5) 6.28 Filling in the exact result for C_V for the rotator system. This is a continuation of problem 6.26. You may find it illuminating to read 6.27 as well. Use Mathematica.

(6) 6.39 More on Earth's (and the moon's) atmosphere. For part (b) add a computation for CO_2 and comment on the lifetime of CO_2 in the atmosphere as compared to nitrogen and hydrogen gas.

(7) 7.3 Deriving the Saha relation. See pages 218-9 on why this is interesting.

(8) **Black hole partition functions II** Continuing with our partition function for the quantum geometry of a black hole for observers near the horizon...

(a) Starting from the 1 geometric particle partition function in problem 6(c) of Guide 8, find the heat capacity for one particle. Explain why the total heat capacity is just N times this result for the whole black hole geometry. Using Mathematica plot the heat capacity (or C_v/Nk) as a function of dimensionless temperature kT/ϵ. Please add the asymptotic value to your plot, if it has one.

(b) By starting with the expectation value for area show that

$$A_H = \langle A \rangle = \frac{8\pi G}{gc^2} \langle E \rangle .$$

(c) As you know these, observers are bathed in thermal Hawking radiation. When the observers' acceleration g is large - which it is when the observers are just outside the horizon - the temperature of the radiation is

$$T_g = \frac{\hbar g}{2\pi c k}.$$

Find the entropy at the temperature T_g. Compare to the average horizon area.

(d) Is Bekenstein's result for black hole entropy

$$S_{BH} = k\frac{A_H c^3}{4G\hbar} = k\frac{A_H}{4\ell_P^2}$$

in your result? It should be! Pause for a moment on the significance of this result: You have derived the macroscopic form for the entropy of a black hole from gravitational statistical mechanics (from a tentative theory of quantum gravity)! This is like deriving the ideal gas law from statistical mechanics for a particle in an ∞-square well.

(e) But let's work out the value of the extra term for these observers. Express the extra term(s) as functions of the area, keeping in mind that this is valid for large black holes for which N is very large. At the observers' temperature T_g,[1]

$$\beta\epsilon = \pi(\ell/\ell_P)^2 \simeq 0.86.$$

What is value of $\ln Z_1$ and the extra term(s) for these observers?

[1] and a certain value for ℓ/ℓ_P