Stat Mech Midterm Information

Fall 2024 v1.5

Material Covered:

- Schroeder Chapters 1, sections 1 - 6, and just the beginning and end of section 7, 2, 3, 4 sections 1 and 2, 5 sections 1 and 2, and 6 sections 1 and 2
- Topics covered in class through October 24 including the house energy flow calculations and the BOE calculation of climate change
- Topics include:
  - Basic thermo: first and second laws, equipartition theorem, "$Q = mc\Delta T$" type problems, heat transfer through conduction and radiation, etc.
  - Heat Engines
  - Thermodynamic free energies and relations
  - Definitions of micro- and macro-states, multiplicity, entropy, temperature, pressure, etc.
  - Einstein solids
  - Basic partition functions
- We **will not** do computations based on chemical reactions on the midterm.
- **Know** the first and second laws of thermodynamics, the definition of the partition function, and the probability $P(s)$.

Midterm Instructions:

Welcome to the Stat Mech midterm! On the logistics side:

- Be sure to have a calculator on hand.
- Other than the test, consult no resources
- You have 75 minutes, maximum, to complete your solutions.
- The weighting of the problems is as shown.
- Your solutions must be entirely your own work.
- Please ask questions, particularly when the problem is not clear!

Handy Relations Please let me know if you see something you would like to have handy...

$$N_A = 6.02 \times 10^{23}$$
$$k = 1.38 \times 10^{-23} \text{ J/K } = 8.62 \times 10^{-5} \text{ eV/K}$$
$$R = 8.32 \times \text{ J/(mol K)}$$
$$h = 6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s}$$
$$\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ J s}$$
$$G = 6.673 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$$
$$c = 2.998 \times 10^8 \text{ m s}^{-1}$$
$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35} \text{ m}$$
$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$
$$\binom{N}{n} = \frac{N!}{n!\,(N-n)!}$$
$$f(x) = f(0) + \left.\frac{df}{dx}\right|_{x=0} x + \frac{1}{2}\left.\frac{d^2f}{dx^2}\right|_{x=0} x^2 + \frac{1}{6}\left.\frac{d^3f}{dx^3}\right|_{x=0} x^3 + \dots \quad (1)$$
$$\cosh(x) = \frac{e^x + e^{-x}}{2}; \;\; \sinh(x) = \frac{e^x - e^{-x}}{2}$$
$$\ln(1+x) \simeq x - \frac{1}{2}x^2$$
$$1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$$
$$1 + x + x^2 + \cdots + x^N = \frac{1 - x^{N+1}}{1-x}$$
$$T \text{ (in K) } = T \text{ (in }^\circ\text{C) } + 273$$
$$1 \text{ atm} = 1.01 \text{ bar } = 1.01 \times 10^5 \text{ Pa}$$
$$kT_{room} \approx \frac{1}{40} \text{ eV}$$
$$kT = \frac{1}{\beta}$$

$$\Omega(N,q) = \binom{q+N-1}{q} \text{ for Einstein solid}$$
$$N! \approx N^N e^{-N} \sqrt{2\pi N}$$
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-a)^2/(2\sigma^2)} \quad (2)$$
$$PV = NkT$$
$$W = -\int PdV$$
$$V^\gamma P = \text{constant for adiabatic } (Q=0) \text{ processes } \gamma = \frac{f+2}{f}$$

$$
\begin{aligned}
U_{thermal} &= N\,f\,\frac{1}{2}kT \\
e &= \frac{\text{benefit}}{\text{cost}} \leq 1 - \frac{T_c}{T_h} \text{ for heat engines} \\
\text{COP } &= \frac{\text{benefit}}{\text{cost}} \leq \frac{T_c}{T_h - T_c} \text{ for heat pumps} \\
V^{\gamma}P &= \text{ constant} \\
S &= k \ln \Omega \\
C_V &= \left(\frac{\partial U}{\partial T}\right)_V \\
H &= U + PV \\
F &= U - TS = -kT \ln Z \\
G &= U + PV - TS = N\mu \\
S &= \left(\frac{\partial F}{\partial T}\right) \\
\Delta S &= \frac{dQ}{T};\, \Delta S = \int \frac{C_V}{T} dT \\
\frac{1}{T} &= \left(\frac{\partial S}{\partial U}\right)_{N,V} \\
P &= T\left(\frac{\partial S}{\partial V}\right)_{N,U} \\
\mu &= -T\left(\frac{\partial S}{\partial N}\right)_{V,U} \\
\frac{dQ}{dt} &= -k_t A \frac{\Delta T}{\Delta x} \\
P &= \sigma A T^4 \\
dU &= TdS - PdV + \mu dN \\
U &= -\frac{\partial \ln Z}{\partial \beta}
\end{aligned}
\tag{3}
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