Topics in Mathematical Physics (PHYS 320): QPS 2 Spring 2019

Welcome to the problem set on Fourier series, Laplace transforms, and series solutions!

- Please submit your solutions in class on Tuesday March 5.
- Please use your notes Mathematica, Wolfram Alpha, Schaum's, and Boas, but no other resources. If you use software please include printouts of your work using the program(s).
- You may not consult any other resources such as the math methods sites on the internet.
- Your solutions must be entirely your own work.
- Please check your results.

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(1) (10 pts.) Using the second shifting property find the Laplace transform of the unit step function

$$u_a(x) = \begin{cases} 0, & x \leq a \\ 1, & x > a \end{cases}$$

Assume $a > 0$.

(2) (10 pts.) Solve $u'' + 4u' + 4u = 0$ for $u(x)$ with initial conditions $u(0) = 1$ and $u'(0) = 0$ using Laplace transforms.

(3) (20 pts.) Solve the initial value problem

$$u''(x) + 0.02u'(x) + 36u(x) = f(x) \text{ for } u(0) = 0 \text{ and } u'(0) = 0$$

where the function $f(x)$ is a periodic function with period 6. On the domain $(0, 6)$ it is given by

$$f(x) = \begin{cases} 6x^2, & 0 \leq x \leq 3 \\ 0, & 3 < x < 6 \end{cases}$$

Use Laplace transforms to obtain a solution. Hint: Try working with just f on the interval $(0, 6)$ first. Then build in the periodicity.

(4) (5 pts.) Find the Wronskian for the "spherical Bessel equation"

$$x^2y'' + 2xy' + (x^2 - \ell(\ell + 1))y = 0.$$

(5) (15 pts.) Solve the ODE

$$u'' + 4x^2u = 0$$

using the series method if $u(0) = 4$ and $u'(0) = -2$. Find an approximate value for $u(2)$.