Math Methods (PHYS 320):19 Spring 2019

Reading: Boas Chapter 12, section 17.

(1) You started this one in Questions 18 - I just include the first (corrected) parts since they had a bad typo. For Thursday start with part e.

(a) Separate the TISE form of Schrödinger's equation (equation 3.22 on page 631) in spherical coordinates assuming that

$$V(r) = \frac{1}{2}\mu\omega^2 r^2.$$

This is a 3D harmonic oscillator. Assume the mass is μ.

(b) Find the angular solutions - much like we just did in class.

(c) When you get to the radial equation it is convenient to define

$$R(r) = \frac{u(r)}{r},\ \rho = \alpha r = \sqrt{\frac{\mu\omega}{\hbar}}r, \text{ and } \lambda = \frac{2E}{\hbar\omega}.$$

Show that the ODE becomes

$$\frac{d^2u}{d\rho^2} - \frac{\ell(\ell+1)}{\rho^2}u - \rho^2 u = -\lambda u.$$

(d) Up to typos this is where you finished for Tuesday.

(e) Use our analysis in asymptopia to motivate the change of dependent variable to

$$u(\rho) = \rho^{\ell+1}e^{-\rho^2/2}f(\rho)$$

(f) Solve for $f(\rho)$ by finding a named special function ODE.

(g) Write the solutions in the form $\psi_{nlm}(r,\theta,\varphi)$ - the 'story so far'.

(h) Find the energy levels E_n.