General Relativity (PHYS 325) PS 3

Spring 2026 v1.0

This third problem set is on special relativity and the initial stages of the calculus of spacetime geometry.

Reading: We have completed Chapter 1.

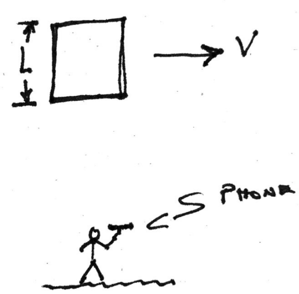
Schutz Chapter 2

Problems:

All numbered problems are from Schutz.

(1) In 1959 Roger Penrose[1] wrote, " ...the appearance of a sphere, no matter how it is moving, is always such as to present a *circular* outline to any observer." The shape seen in a photograph of a relativistically moving sphere (and what we actually see) is still a sphere.

Let's show a related result for a cube. If a photograph is taken of a cube, with edge length L, moving uniformly with a speed v and direction parallel to the film (or a CCD detector in a phone), it appears rotated (and warped) rather than contracted in the image. Here's the set-up:

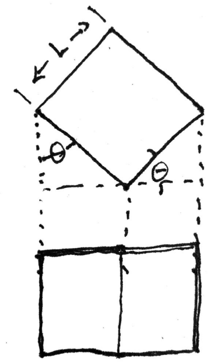


(a) What is the length of the cube in the direction of the velocity? Explain your answer briefly and sketch the shape. (What "is" is the shape on one surface of simultaneity.)

(b) Let's find the image assuming the cube is far from the phone, i.e. L is very small relative to the distance between the phone and the cube. This means that every light ray arriving at the phone from the cube is essentially parallel. Light emitted from the far corner takes L/c ($= L$ when $c = 1$) additional time to arrive at the phone. Explain why it would appear displaced Lv from the image of the face closest to the phone. Sketch the image of the cube with lengths of the two visible faces. (It may be helpful to picture this as a wire frame of a cube.)

(c) Consider a cube at rest rotated by θ as shown

[1]In case you are wondering about all these references to Roger's work, I recently paged through volume 1 of the collected works of Roger Penrose. Why? I recently was surprised to discover in this volume that Penrose's Spin Geometry Theorem was essentially proved roughly a decade earlier than I had thought - as an appendix to his Adam's Prize essay.



I've also included the image - what we see. Based on the last two parts of this problem, show that the image of the moving cube is rotated. Find the angle of rotation θ.

(d) If $v = 0.8$ find the width of the image and the angle of apparent rotation.

This shows the potential problem with using terms like "see", "appear," and "look" when describing relativistic effects. They are different than "is".

(2) A steel cable connects two rocket ships at rest with respect to each other. The cable snaps if it is stretched by more than 1%. The rockets accelerate uniformly so that after the engines fire their velocities at each moment of time, as measured in the ground frame, are always equal. Eventually the cable snaps.

(a) Explain why this happens. Write up explanations in both the ground frame and each of the rockets' frames.

(b) In the ground frame how fast are the rockets moving when the cable snaps?

(c) Do there exist reference frames in which it is possible to attach a new cable to the two (moving) rockets at one moment in time (or surface of simultaneity) such that the cable is not stretched? If so what are these frames? If not, why is this not possible?

(3) 2.14 An example following up on 1.20.

(4) 2.15 Working with 4-velocities

(5) Suppose you have two 4-vectors

$$w^\alpha \to (-2, 0, 0, 1) \text{ and } u^\beta \to (5, 0, 3, 4).$$

(a) Is u^β timelike, null or spacelike?

(b) Find $\vec{w} \cdot \vec{u}$.

(6) In the LHC at CERN protons can travel in opposite directions around a circular ring of approximately 27 km in radius at an energy of 7 TeV apiece.

(a) How close are the particles speeds relative to light?

(b) How many turns around the ring would a proton make in 10 hours?

(7) 2.12 parts (a) and (c) only. Practice with Lorentz transformations

(8) (2 pts.) 2.19 Uniform acceleration and a trip to the center of our galaxy