General Relativity (PHYS 325) PS 4

Spring 2026 v1.0

This fourth problem set has a bit on 4-vector notation, particle kinematics, and vector differentiation. For your amusement...

> If γmv yields frustration
> And similar E, irritation,
> Just ditch all the v's,
> And use (won't you please)
> The Very Important Relation.
> - D. Morin.

The Very Important Relation is none other than $E^2 = p^2 + m^2$.

Reading:
Schutz Chapter 3

Problems: Solutions due by 11 PM on Thursday, February 19
All numbered problems are from Schutz.

(1) A 'warped sphere' has metric

$$ds^2 = a^2\left(d\theta^2 + f^2(\theta)d\varphi^2\right)$$

where $f(\theta) = \sin\theta(1 + \epsilon\sin^2(\theta))$ and $0 < \epsilon < 1$.
   (a) What is the interpretation of a? Hint: Set $\epsilon = 0$.
   (b) Find the circumference of the equatorial ($\theta = \pi/2$) great circle C_e.
   (c) Find the circumference of the polar ($\theta = 0$) great circle C_p.
   (d) Earth has $\Delta C = C_e - C_p \simeq 21$ km and an average polar radius of 6357 km. Find ϵ for Earth.

(2) 2.21 Continuing our work on acceleration

(3) 2.30 part (b) only: Finding observed energy

(4) Hot Potato-Cold Potato: Suppose you have two identical 0.52 kg potatoes at room temperature. You put one in the oven to give it 150 J of energy (it is now Hot). Before eating the Hot potato, you accelerate the two potatoes with identical forces. Do the potatoes accelerate at the same rate? If so, find the ratio of potato accelerations. If not, explain how the thermal energy remains free of inertia.

(5) Using the Very Important Relation: A particle of mass M decays into two identical particles of mass m. In the frame $\mathcal{O}$ particle 1 is emitted at 90 degrees from the original particle's velocity. Let's call the 4-momenta of the particles p_0, p_1, and p_2. We want to find the energies of the products E_1 and E_2.
   (a) Write down the squares of the three 4-momenta.
   (b) Energy and momentum conservation is expressed as $p_0 = p_1 + p_2$. Square $p_0 - p_1 = p_2$ to obtain E_1. (Don't forget the cross term.)
   (c) Find E_2 from energy conservation.

(6) 3.4 parts (b) and (c)

(7) 3.10 Checking that the derivative form of the Lorentz transformations, equation 3.18, makes sense

(8) 3.18 In part (a) only lower the index for vectors A, B, and C. In part (b) raise the index for p, q and r only.