General Relativity (PHYS 325) PS 5

Spring 2026 v1.0

This week there is a mix of 4-vectors, 1-forms, and metric calculations, a SR result, and one question on particle kinematics. The reading in Chapter 5 will probably take you a little ahead of where we'll be in class, but this is intensional so you can see where we are headed - the mathematics of curvature.

Reading:

Schutz Chapter 4 sections 1 - 4 (skim sections 5 and 7 if you have had some thermodynamics) and pages 96-9

Schutz Chapter 5 sections 1 - 4 (Feel free to read section 5.4 but we will not dwell on non-coordinate bases.)

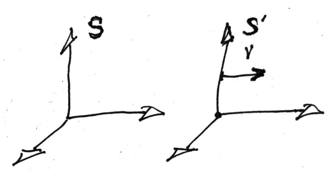
Problems: Solutions due by 11 PM on Thursday, February 26

All numbered problems are from Schutz.

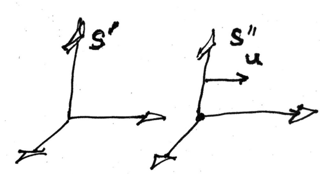
(1) 3.14 For understanding the outer product notation.

(2) 3.30 (2 pts. for length) If you have doubts about earlier parts then please ask.

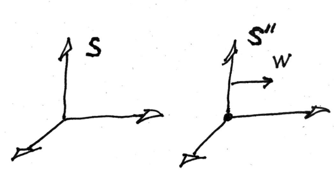
(3) Suppose you have *three* reference frames, $\mathcal{S}$, $\mathcal{S}'$, and $\mathcal{S}$". They all share a common "x" axis. The primed reference frame moves at v along the x-axis in the frame $\mathcal{S}$ as shown in this spatial view:



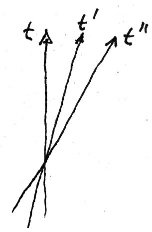
The double-primed reference frame $\mathcal{S}$"moves at u along the x'-axis in the frame $\mathcal{S}'$:



And, of course, the double-primed reference frame $\mathcal{S}$"moves at some speed along the x-axis in the frame $\mathcal{S}$:



Let's call this speed w as shown. Here's a view in spacetime



What is w in terms of v and u?

(a) You can find the answer this way: Let's denote the coordinate of the $\mathcal{S}'$ origin in the frame $\mathcal{S}$ to be $x_{\mathcal{S}'}$. Let's assume they all synchronize their clocks as they pass. Thus,
$$v = \frac{x_{\mathcal{S}'}}{t}.$$
Similarly, if the coordinate of the $\mathcal{S}''$ origin in the frame $\mathcal{S}$ is $x_{\mathcal{S}"}$ then
$$w = \frac{x_{\mathcal{S}''}}{t}.$$
By using the inverse Lorentz transformation from $\mathcal{S}$ to $\mathcal{S}'$ (why inverse?) derive the relation between the three speeds.

(b) What is w when $v = u = 0.75$?

(c) What is the maximum value for w? Explain or prove your result.

(d) Check that the result reduces to the Galilean $w = v + u$ for relatively small speeds.

This is a key result of special relativity.

(4) When considering particle collisions it is often handy to work in the frame in which the total three-momentum vanishes. This is called the center of momentum frame.

A particle of type "A", with mass m_A, crashes into a particle of type "B", with mass m_B, which is at rest in the frame of the lab. This collision produces a shower of particles of type "C", with masses $m_1, m_2, \ldots$. At the threshold energy for this process, the particles of type "C" go ... *absolutely nowhere* in the center of momentum frame; they are at rest (which makes sense for a threshold, right?).

(a) Write down the conservation of 4-momentum for this collision.

(b) Square this equation to obtain the invariant before and after the collision.

(c) By going to the lab frame before the collision and the center of momentum frame after the collision show that the threshold energy of this process is
$$E_A = \frac{M^2 - m_A^2 - m_B^2}{2m_B},$$
where $M = m_1 + m_2 + \ldots$ is the sum of the masses of the particles of type "C".

(5) 4.4